Appendix

A Proofs of Results In-Text

Lemma 1. *Given incentive and disincentive policies of R and P, the share of the population choosing $a = 1$ is $1 - F(-R - P)$, while the share of the population choosing $a = 0$ is $F(-R - P)$.*

Proof of Lemma 1. Given a policy (P, R) and the assumption that i chooses 1 when indifferent, a member of the population i optimally chooses $a_i = 1$ if $v_i \geq -R - P$, and $a_i = 0$ otherwise.

The collection of members of the population choosing $a_i = 1$ are given by the set $\{i | v_i \geq -R - P\}$, representing a share given by $1 - F(-R - P)$. ■

Lemma 2. *It is never optimal to use strictly positive levels of both punishments and rewards.*

Proof of Lemma 2. Denote the share of the population with $v_i \geq 0$ by Γ_0. Recall that $C_p(\cdot)$ and $C_r(\cdot)$ are increasing in their arguments, respectively, $1-\Gamma$ and Γ. As such, there either exists a share of the population taking $a = 1$, denote it $\hat{\Gamma} \in [\Gamma_0, 1]$, such that $C_p(1-\hat{\Gamma}) = C_r(\hat{\Gamma})$, or it must be true that $C_p(1-\Gamma_0) < C_r(\Gamma_0)$ or $C_p(0) > C_r(1)$. In the latter two possibilities, the cheapest policy instruments at any share of the population choosing $a = 1$ are, respectively, disincentives and incentives.

Suppose that the first case obtains, however, where $C_p(\hat{\Gamma}) = C_r(1 - \hat{\Gamma})$ for some $\hat{\Gamma} \in [\Gamma_0, 1]$. For all lower shares taking $a = 1$, $\Gamma < \hat{\Gamma} \Rightarrow C_r(\Gamma) < C_p(1 - \Gamma)$, so rewards are the cheaper type of policy with which to attain a given level of compliance. Conversely, for higher shares of the population taking $a = 1$, $\Gamma > \hat{\Gamma} \Rightarrow C_p(1-\Gamma) < C_r(\Gamma)$, so punishments are the cheaper type of policy with which to attain a given level of compliance. At $\hat{\Gamma}$, either type of policy entails the same administrative cost, but only one should be used so that costs are not incurred twice to achieve the compliance of the $\hat{\Gamma}$ share of the population. To induce any share of the population to choose $a = 1$, then, the optimal policy will entail only one type of policy. ■

As discussed in text, we consider a choice of policy $\tau \in \{p, r, n\}$, with p denoting the use of punishments, r denoting the use of rewards, and n denoting the absence of a policy intervention, as well as a choice of the share of the population choosing $a = 1$ ex post. Lemma 5 demonstrates the equivalence of the (Γ, τ) formulation to the (P, R) formulation. In particular, it demonstrates the equivalence of the latter formulation to the approach of finding the optimal (Γ, θ) and then comparing to (Γ_0, n).

Lemma 5. *Let $X(\Gamma) = -F^{-1}(1 - \Gamma)$. If (Γ^*, τ^*) is a member of the set of optimal choices under the (Γ, τ)*

formulation, then there exists a member of the set of optimal choices under the (P, R) formulation given by

$$(P^*, R^*) = \begin{cases} (X(\Gamma^*), 0), & \tau^* = p \\ (0, X(\Gamma^*)), & \tau^* = r \\ (X(\Gamma_0), X(\Gamma_0)) = (0, 0), & \tau^* = n. \end{cases}$$

If (P^*, R^*) is a member of the set of optimal choices under the (P, R) formulation, then there exists a member of the set of optimal choices under the (Γ, τ) formulation given by

$$(\Gamma^*, \tau^*) = \begin{cases} (1 - F(-P^* - R^*), p), & P^* > 0, R^* = 0 \\ (1 - F(-P^* - R^*), r), & P^* = 0, R^* > 0 \\ (1 - F(-P^* - R^*), n), & P^* = R^* = 0. \end{cases}$$

Proof of Lemma 5. Define $X := P + R$. Given Lemma 1, there exists a one-to-one and onto relation between Γ and the value of X in equilibrium, given by $\Gamma(X) = 1 - F(-X) \Leftrightarrow X(\Gamma) = -F^{-1}(1 - \Gamma)$. Recall that Lemma 2 establishes that at most one of P or R will be strictly greater than zero. As such, when considering the optimal intervention, we may use θ to describe the type of policy in an active intervention, where $\theta = 1$ if $\tau = p$, and $\theta = 0$ if $\tau = r$. In the notation of equation (1), $\pi = 1 - \rho$ in the optimal intervention by Lemma 2, so $\theta = \pi$. As in equation (3), set

$$(\Gamma^*, \theta^*) := \underset{\Gamma \geq \Gamma_0, \theta \in \{0,1\}}{\arg\max} \; W(\Gamma) - \int_0^\Gamma X(\hat{\Gamma}) d\hat{\Gamma} - (1 - \theta) C_r(\Gamma) - \theta C_p(1 - \Gamma).$$

The main difference between the two maximization problems is the domains, so we show that the solutions to each problem lie in the domain of the other problem. By Lemma 2, the optimal policy, (P^*, R^*) lies in the set $\{(P, 0) | P \geq 0\} \cup \{(0, R) | R \geq 0\}$. By the independence of irrelevant alternatives axiom, we know that eliminating the set of policies $\{(P, R) | P > 0, R > 0\}$ and maximizing over $(P, R) \in \{(P, 0) | P \geq 0\} \cup \{(0, R) | R \geq 0\}$ yields the same solutions as maximizing over $(P, R) \in \mathbb{R}_+^2$ would yield. There exists a one-to-one and onto relation between $(P, R) \backslash (0, 0)$ and $(\Gamma, \theta) \backslash (\Gamma_0, \theta)$, where $(P, R) = (\theta \cdot X(\Gamma), (1 - \theta) \cdot X(\Gamma))$. If (P^*, R^*) maximizes equation 3 and $P^* + R^* > 0$, then $(\Gamma^*, \theta^*) = (1 - F(-R^* - P^*), \pi(P^*))$.

Note, however, that it need not be the case that (Γ^*, θ^*) as defined in (3) with $\Gamma^* > \Gamma_0$ implies $(P^*, R^*) = (\theta^* \cdot X(\Gamma^*), (1 - \theta^*) \cdot X(\Gamma^*))$, since the (Γ, θ) formulation imposes the use of rewards or punishments. This is akin to a zero-dollar fine or subsidy, in essence incurring the administrative cost without any actual transfer taking place. If $(P^*, R^*) = (0, 0)$ and $(\Gamma^*, \theta^*) = (\Gamma_0, \theta^*)$, then (Γ_0, n) will be optimal in the unconstrained formulation allowing $\tau \in \{n, p, r\}$, as it achieves the same compliance at no cost. If $(P^*, R^*) = (0, 0)$ and (Γ^*, θ^*) entails $\Gamma^* > 0$, then (Γ_0, n) will again be at least as preferred in the unconstrained problem, otherwise $(P^*, R^*) = (\theta^* \cdot X(\Gamma^*), (1 - \theta^*) \cdot X(\Gamma^*))$. ■

Proposition 1. *Consider $W, \hat{W}$, and suppose $\hat{W}' > W'$ pointwise. Then (Γ^*, θ^*) is weakly larger under $\hat{W}$ than under W.*

Proof of Proposition 1. If the utilitarian objective function has monotone comparative statics in some exogenous variable z, then an increase in z will lead to an increase in the optimal policy intervention, i.e., a weakly larger share of the population choosing $a = 1$ *ex post* and either switching from using rewards to using punishments or the continued use of punishments. The definition below restates for convenience a notion of complementarity used throughout, viz., increasing differences. The next result then establishes that the utilitarian social planner's objective function displays increasing differences in the choice variables.

Definition 3 (Increasing Differences)**.** *Consider a single-valued objective function $U : Y \times Z \to \mathbb{R}$. If, for all $y, \hat{y} \in Y$ where $y < \hat{y}$ and for all $z, \hat{z} \in Z$ where $z < \hat{z}$, $U(\hat{y}, \hat{z}) - U(\hat{y}, z) > U(y, \hat{z}) - U(y, z)$, then the function U has increasing differences in the pair of arguments (y, z).*[17] *If the inequality holds weakly, refer to it as non-decreasing, and if it holds with $<$ $(\leq)$, then decreasing (non-increasing).*

Lemma 6. *The objective function in equation* (3) *has increasing differences in* (Γ, θ).

Proof of Lemma 6. Let

$$U_{SP}(\Gamma, \theta) = W(\Gamma) - \int_0^{\Gamma} X(\hat{\Gamma}) d\hat{\Gamma} - (1 - \theta) C_r(\Gamma) - \theta C_p(1 - \Gamma).$$

We show $U_{SP}(\Gamma, \theta)$ displays increasing differences in (Γ, θ) by demonstrating that $U_{SP}(\hat{\Gamma}, \hat{\theta}) - U_{SP}(\hat{\Gamma}, \theta) \geq U_{SP}(\Gamma, \hat{\theta}) - U_{SP}(\Gamma, \theta)$, for all $\hat{\Gamma} > \Gamma$ and $\hat{\theta} > \theta$ (which implies $\hat{\theta} = 1$ and $\theta = 0$).

$$\begin{aligned} U_{SP}(\hat{\Gamma}, \hat{\theta}) - U_{SP}(\hat{\Gamma}, \theta) &= C_r(\hat{\Gamma}) - C_p(1 - \hat{\Gamma}) \\ U_{SP}(\Gamma, \hat{\theta}) - U_{SP}(\Gamma, \theta) &= C_r(\Gamma) - C_p(1 - \Gamma) \end{aligned}$$

The first line is greater than the second if $C_p(1 - \Gamma) - C_p(1 - \hat{\Gamma}) \geq C_r(\Gamma) - C_r(\hat{\Gamma})$. The right-hand side is negative, as C_r is increasing in Γ. The left-hand side is positive as C_p is increasing in $1 - \Gamma$ and $\hat{\Gamma} > \Gamma \Rightarrow$

[17] Equivalently, the function U has increasing differences in the pair of arguments (y, z) if the incremental return $U(\hat{y}, \cdot) - U(y, \cdot)$ is increasing in z (or, alternatively, if the incremental return $U(\cdot, \hat{z}) - U(\cdot, z)$ is increasing in y).

$1 - \Gamma > 1 - \hat{\Gamma}$. As such, $U_{SP}(\hat{\Gamma}, 1) - U_{SP}(\Gamma, 1) \geq U_{SP}(\hat{\Gamma}, 0) - U_{SP}(\Gamma, 0)$, which was to be shown. ■

Lemma 6 provides the foundation for applying the theory of monotone comparative statics.

Definition 4 (Monotone Comparative Statics)**.** *Consider a single-valued objective function $U : Y \times Z \to \mathbb{R}$, where $z \in Z$ is an exogenous parameter. Let $Y = \times_{j=1}^{n} Y_j$, such that $(y_1, \ldots, y_n) \in Y$ is a vector of endogenous (choice) variables. If U displays non-decreasing differences in all pairs $(y_j; z)$ – with increasing differences in at least one pair – as well as non-decreasing differences in all pairs (y_j, y_k), $j \neq k$, then U displays monotone comparative statics with respect to increases in z, and an increase in z leads to a weak increase in the optimal $(y_1, \ldots, y_n)$.*[18]

Lemma 7. *Let z represent an exogenous element of the model, with a given notion of increasing for $z \in Z$. If the objective function in equation (3) has non-decreasing differences in (Γ, z) as well as in (θ, z) – with increasing differences in at least one of the pairs – then it has monotone comparative statics in z.*

Proof of Lemma 7. Letting $z = t$, $(\Gamma, \theta) = y$, $[0, 1] \times \{p, r\} = Y$, and $[\Gamma_0, 1] = S$, we recognize that the product set Y is a lattice, so we may then apply Theorem 5 from Milgrom & Shannon (1994, p. 164):[19]

> Let Y be a lattice, T a partially ordered set, and $g : Y \times T \to \mathbb{R}$. If $g(y, t)$ is supermodular in y and has increasing differences in $(y; t)$, then $\arg\max_{y \in S} g(y, t)$ is monotone nondecreasing in (t, S).[20]

Lemma 6 demonstrated increasing differences in the choice variables – satisfying the supermodularity in y in theorem quoted above. The premise of Lemma 7 supposes increasing differences between an exogenous parameter and each of the choice variables – satisfying the increasing differences in $(y; t)$ to which the quoted theorem refers.[21] ■

[18] All increases in variables are with respect to a given partial ordering. If the set of optimal $(y_1, \ldots, y_n)$ is not a singleton, then the increase in the optima is with respect to the strong set ordering.

[19] Ashworth & Bueno de Mesquita (2006, p. 221), among others, also provide statements of the conditions of complementarity under which monotone comparative statics hold without further parameterization. Specifically, and provided that the choice set is a product set, we seek to show each pair of arguments of the utility function has increasing differences. If it can then be shown that U_{SP} has increasing differences with respect to an exogenous parameter and each of the choice variables, then we may conclude that an increase in that parameter leads to an increase (in the strong set order) in the (set of) optimal of optimal policy intervention(s), (Γ^*, θ^*). We need not worry about indirect effects. The pairwise complementarity of parameters and choice variables ensures that any indirect effects only enhance the direct effects.

[20] Attributed by the authors to Topkis (1978).

[21] Relevant partial orderings on the parameter space are provided below.

It remains to be shown, then, whether the objective function in equation (3) has increasing differences in $(\Gamma, \theta; W(\cdot))$, with the partial ordering for W supplied in the Proposition. To do so, we must demonstrate increasing differences in each choice variable-parameter pair. For each parameter, we adopt a mix of techniques. To show increasing differences in θ and the parameter, we compare the incremental return of a discrete increase in the parameter under $\theta = r$ and $\theta = p$. This follows the approach taken to show the increasing differences of (Γ, θ) in the proof of Lemma 6.

We proceed differently to show increasing differences in Γ and the parameter. For example, for $W(\cdot)$, we examine $\frac{\partial}{\partial \Gamma}(U_{SP}(\Gamma, \theta; \hat{W}) - U_{SP}(\Gamma, \theta; W))$. If that quantity is positive, increasing differences may be inferred. Note that

$$\frac{\partial}{\partial \Gamma} U_{SP}(\Gamma, \theta; W(\cdot), F(\cdot), C_p(\cdot), C_r(\cdot)) = W'(\Gamma) + F^{-1}(1-\Gamma) - (1-\theta)C_r'(\Gamma) - \theta C_p'(1-\Gamma).$$

Partially order the set of functions $\{W(\cdot) | W' \geq 0\}$ with the rule:

$$\hat{W} > W \Leftrightarrow \hat{W}' > W', \forall \Gamma.$$

$(\Gamma; W):\ \frac{\partial}{\partial \Gamma}(U_{SP}(\Gamma, \theta; \hat{W}) - U_{SP}(\Gamma, \theta; W)) = \hat{W}'(\Gamma) - W'(\Gamma) > 0$

$(\theta; W):\ U_{SP}(\Gamma, \hat{\theta}; \hat{W}) - U_{SP}(\Gamma, \theta; \hat{W}) - [U_{SP}(\Gamma, \hat{\theta}; W) - U_{SP}(\Gamma, \theta; W)] = 0$

Since, U_{SP} has increasing differences in $(\Gamma; W)$ and non-decreasing differences in $(\theta; W)$, we may conclude the optimal policy intervention displays monotone comparative statics in W. ■

Proposition 2. *Consider $F, \hat{F}$, and suppose for all $v \in (\underline{v}, \overline{v})$, $\hat{F}(v) < F(v)$. Then (Γ^*, θ^*) is weakly larger under $\hat{F}$ than under F.*

Proof of Proposition 2. The proof follows the same format as the proof for Proposition 1, up to the demonstration of increasing differences between the choice variables and F.

$(\Gamma; F):\ \frac{\partial}{\partial \Gamma}(U_{SP}(\Gamma, \theta; \hat{F}) - U_{SP}(\Gamma, \theta; F)) = \hat{F}^{-1}(1-\Gamma) - F^{-1}(1-\Gamma) \geq 0$, ($>$ for $\Gamma \in [0,1]$)

$(\theta; F):\ U_{SP}(\Gamma, \hat{\theta}; \hat{F}) - U_{SP}(\Gamma, \theta; \hat{F}) - [U_{SP}(\Gamma, \hat{\theta}; F) - U_{SP}(\Gamma, \theta; F)] = 0$

Since, U_{SP} has increasing differences in $(\Gamma; F)$ and non-decreasing differences in $(\theta; F)$, we may conclude the optimal policy intervention displays monotone comparative statics in F.

Should a change in a parameter affect the constraint set for the the maximization problem, Theorem 5 from Milgrom & Shannon (1994) provides the condition under which we may still infer monotone comparative statics. Specifically, as long as the constraint set is increasing (in the strong set order) and the strict single crossing property is satisfied in the parameter (implied by the increasing differences above), we may proceed as before. Since $1 - \hat{F}(0) > 1 - F(0)$, the constraint set $[\Gamma_0, 1]$ is increasing in the strong set ordering. ■

Proposition 3. *Consider $C_\tau, \hat{C}_\tau$, $\tau = p, r$, and suppose $\hat{C}'_\tau < C'_\tau$ pointwise. Absent additional assumptions, (Γ^*, θ^*) may be larger or smaller under $\hat{C}_\tau$ than under C_τ.*

Proof of Proposition 3. The proof again follows the same format as the proof for Proposition 1, up to the demonstration of increasing differences between the choice variables and C_τ, $\tau = p, r$. Note that since we have not assumed any change to the fixed costs, $\hat{C}_r(\Gamma) < C_r(\Gamma)$ and $\hat{C}_p(1-\Gamma) < C_p(1-\Gamma)$, $\Gamma \in (0, 1)$.

$$(\Gamma, C_p): \ \frac{\partial}{\partial\Gamma}(U_{SP}(\Gamma, \theta; \hat{C}_p) - U_{SP}(\Gamma, \theta; C_p)) = \theta(\hat{C}'_p(1-\Gamma) - C'_p(1-\Gamma)) < 0$$

$$(\theta, C_p): \ U_{SP}(\Gamma, 1; \hat{C}_p) - U_{SP}(\Gamma, 0; \hat{C}_p) - [U_{SP}(\Gamma, 1; C_p) - U_{SP}(\Gamma, 0; C_p)] = -\hat{C}_p(1-\Gamma) + C_p(1-\Gamma) > 0.$$

$$(\Gamma, C_r): \ \frac{\partial}{\partial\Gamma}(U_{SP}(\Gamma, \theta; \hat{C}_r) - U_{SP}(\Gamma, \theta; C_r)) = (1-\theta)(C'_r(\Gamma) - \hat{C}'_r(\Gamma)) > 0$$

$$(\theta, C_r): \ U_{SP}(\Gamma, 1; \hat{C}_r) - U_{SP}(\Gamma, 0; \hat{C}_r) - [U_{SP}(\Gamma, 1; C_r) - U_{SP}(\Gamma, 0; C_r)] = \hat{C}_r(\Gamma) - C_r(\Gamma) < 0$$

In neither the case of C_p nor C_r does the utilitarian's welfare function display increasing (or decreasing) differences between the parameter and *both* of the choice variables, which was to be shown. ■

Lemma 3. *All members of the population prefer the use of either an incentive or a disincentive policy – but not both – to achieve any given level of compliance.*

The proof follows the exact same logic as the proof of Lemma 2.

Lemma 4 *A majority of the population will prefer the median voter's most-preferred policy intervention, $(\Gamma^{**}, \theta^{**})$, over any given alternative $(\Gamma, \theta^*(\Gamma))$, and a majority of the population will choose a^{**}_{MV} under the policy.*

Proof of Lemma 4. We wish to invoke Theorem 1 from Gans & Smart (1996), which the authors summarize as stating "that when preference profiles are single-crossing, the median voters on the order of $[v \in [\underline{v}, \overline{v}]]$ are decisive in all majority elections between pairs of alternatives $[\Gamma, \hat{\Gamma} \in [0, 1]]$" (p. 222, substitutions in brackets). We must demonstrate, then, that preferences over Γ are single-crossing in v. Of course, a could change as a function of changes in both v and Γ, and θ could change as a function of Γ. The latter is assumed to take its optimal value for a given Γ to reduce the dimensionality of the choice problem, as discussed in text, while sequential rationality ensures the former will take its optimal value for a given Γ and v.

Consider $v' > v$ and $\Gamma' > \Gamma$. Supposing

$$W(\Gamma') - (a_i^*(\Gamma', v) - \Gamma') \cdot F^{-1}(1-\Gamma') - (1-\theta^*(\Gamma'))C_r(\Gamma') - \theta^*(\Gamma') \cdot C_p(1-\Gamma') + a_i^*(\Gamma', v) \cdot v$$
$$\geq$$
$$W(\Gamma) - (a_i^*(\Gamma, v) - \Gamma) \cdot F^{-1}(1-\Gamma) - (1-\theta^*(\Gamma))C_r(\Gamma) - \theta^*(\Gamma) \cdot C_p(1-\Gamma) + a_i^*(\Gamma, v) \cdot v,$$

it remains to be shown that

$$W(\Gamma') - (a_i^*(\Gamma', v') - \Gamma') \cdot F^{-1}(1-\Gamma') - (1-\theta^*(\Gamma'))C_r(\Gamma') - \theta^*(\Gamma') \cdot C_p(1-\Gamma') + a_i^*(\Gamma', v') \cdot v'$$
$$\geq$$
$$W(\Gamma) - (a_i^*(\Gamma, v') - \Gamma) \cdot F^{-1}(1-\Gamma) - (1-\theta^*(\Gamma))C_r(\Gamma) - \theta^*(\Gamma) \cdot C_p(1-\Gamma) + a_i^*(\Gamma, v') \cdot v'.$$

Increasing differences follows if

$$a_i^*(\Gamma', v')(v' - F^{-1}(1-\Gamma')) - a_i^*(\Gamma, v')(v' - F^{-1}(1-\Gamma)) \geq a_i^*(\Gamma', v)(v - F^{-1}(1-\Gamma')) - a_i^*(\Gamma, v)(v - F^{-1}(1-\Gamma)). \tag{7}$$

The expression $a_i^*(\Gamma, v)(v - F^{-1}(1-\Gamma))$ is increasing in Γ and v, so increasing differences clearly obtain. The discrete and determinate nature of the choice a^*, however, makes it worthwhile and possible to examine all of the cases carefully. Further, the exercise demonstrates that strict increasing differences do not obtain.

From $v' - F^{-1}(1-\Gamma') > v - F^{-1}(1-\Gamma'), v' - F^{-1}(1-\Gamma) > v - F^{-1}(1-\Gamma)$, six cases follow:

1. $0 > v' - F^{-1}(1-\Gamma') > v - F^{-1}(1-\Gamma)$
2. $v' - F^{-1}(1-\Gamma') \geq 0 > v - F^{-1}(1-\Gamma)$
   (a) $0 > v - F^{-1}(1-\Gamma'), v' - F^{-1}(1-\Gamma)$
   (b) $v' - F^{-1}(1-\Gamma) \geq 0 > v - F^{-1}(1-\Gamma')$
   (c) $v - F^{-1}(1-\Gamma') \geq 0 > v' - F^{-1}(1-\Gamma)$
   (d) $v - F^{-1}(1-\Gamma'), v' - F^{-1}(1-\Gamma) \geq 0$
3. $v' - F^{-1}(1-\Gamma') > v - F^{-1}(1-\Gamma) \geq 0$

Equation (7) reduces, by case, to:

1. $0 \cdot (v' - F^{-1}(1-\Gamma')) - 0 \cdot (v' - F^{-1}(1-\Gamma)) \geq 0 \cdot (v - F^{-1}(1-\Gamma')) - 0 \cdot (v - F^{-1}(1-\Gamma))$, which holds weakly because $0 \geq 0$.

2. (a) $(v' - F^{-1}(1-\Gamma')) - 0 \cdot (v' - F^{-1}(1-\Gamma)) \geq 0 \cdot (v - F^{-1}(1-\Gamma')) - 0 \cdot (v - F^{-1}(1-\Gamma))$, which holds strictly because $v' - F^{-1}(1-\Gamma') > 0$.

   (b) $(v' - F^{-1}(1-\Gamma')) - (v' - F^{-1}(1-\Gamma)) \geq 0 \cdot (v - F^{-1}(1-\Gamma')) - 0 \cdot (v - F^{-1}(1-\Gamma))$, which holds strictly because $F^{-1}(1-\Gamma) > F^{-1}(1-\Gamma')$.

   (c) $(v' - F^{-1}(1-\Gamma')) - 0 \cdot (v' - F^{-1}(1-\Gamma)) \geq (v - F^{-1}(1-\Gamma')) - 0 \cdot (v - F^{-1}(1-\Gamma))$, which holds strictly because $v' > v$.

   (d) $(v' - F^{-1}(1-\Gamma')) - (v' - F^{-1}(1-\Gamma)) \geq (v - F^{-1}(1-\Gamma')) - 0 \cdot (v - F^{-1}(1-\Gamma))$, which holds strictly because $0 > v - F^{-1}(1-\Gamma)$

3. $(v' - F^{-1}(1-\Gamma')) - (v' - F^{-1}(1-\Gamma)) \geq (v - F^{-1}(1-\Gamma')) - (v - F^{-1}(1-\Gamma))$, which holds weakly because $0 \geq 0$.

This establishes non-strict increasing differences between Γ and v. ■

Proposition 4. *If the majority-preferred policy intervention does (resp., does not) induce a majority of the population to choose $a = 1$ instead of $a = 0$, then the level of overall compliance will be greater (resp., less) than in the utilitarian optimal policy, regardless of the type of policy used.*

Proof of Proposition 4. Define

$$U^1_{MV}(\Gamma, \theta) = W(\Gamma) + (1-\Gamma) \cdot (-F^{-1}(1-\Gamma)) - (1-\theta) \cdot C_r(\Gamma) - \theta \cdot C_p(1-\Gamma) + v_{MV}$$

and

$$U^0_{MV}(\Gamma, \theta) = W(\Gamma) + \Gamma \cdot F^{-1}(1-\Gamma) - (1-\theta) \cdot C_r(\Gamma) - \theta \cdot C_p(1-\Gamma).$$

Viewing the median voter's action a_{MV} as a parameter, it follows that the (set of) majority-preferred intervention(s) when $a^{MV} = 1$ is greater than the (set of) majority-preferred intervention(s) when $a^{MV} = 0$. To demonstrate that these interventions "sandwich" the utilitarian-optimal interventions, consider the first derivatives with respect to Γ:

$$\begin{array}{rcll} \frac{\partial U^1_{MV}}{\partial \Gamma} & = & W' + \frac{1-\Gamma}{f(F^{-1}(1-\Gamma))} + F^{-1}(1-\Gamma) - C'_\theta & \\ & > & & \\ \frac{\partial U_{SP}}{\partial \Gamma} & = & W' + F^{-1}(1-\Gamma) - C'_\theta & , \forall \Gamma \in [\Gamma_0, 1]. \\ & > & & \\ \frac{\partial U^0_{MV}}{\partial \Gamma} & = & W' - \frac{\Gamma}{f(F^{-1}(1-\Gamma))} + F^{-1}(1-\Gamma) - C'_\theta & \end{array}$$

Viewed in the context of Proposition 1, were we to define $\hat{W}' = W' + \frac{1-\Gamma}{f(1-\Gamma)}$ and $\hat{\hat{W}}' = W' - \frac{\Gamma}{f(1-\Gamma)}$, we would find $\hat{W}' \geq W'$ and $W' \geq \hat{\hat{W}}'$ pointwise. Hence, it is as though the utilitarian's objective function maximizes over a higher (lower) marginal benefit for the $a^{MV} = 1$ ($a^{MV} = 0$) case. The monotone comparative statics established in 1 imply:

$$\arg\max_{(\Gamma,\theta)} U^1_{MV} \geq \arg\max_{(\Gamma,\theta)} U_{SP} \geq \arg\max_{(\Gamma,\theta)} U^0_{MV}.$$

Since a^{**}_{MV} also indicates the action a majority of the population will take, a majority complying will favor interventions with greater than optimal compliance, while a majority not complying will favor interventions with lower than optimal compliance. ■

Proposition 5. *Consider $W, \hat{W}$, and suppose $\hat{W}' > W'$ pointwise. Then $(\Gamma^{**}, \theta^{**})$ and a^{**}_{MV} are weakly larger under $\hat{W}$ than under W.*

Proof of Proposition 5. Recall that $(\Gamma^{**}, \theta^{**})$ denotes the median voter's set of optimal policy interventions and a^{**}_{MV} the optimal action associated with each policy intervention, where $a^{**}_{MV} := a^*_{MV}(\Gamma^{**}, v_{MV})$, $\theta^{**} := \theta^*(\Gamma^{**})$, and Γ^{**} is as in 5 (the set of maximizers of the equation below).

$$U_{MV}(\Gamma) = W(\Gamma) - (a^*_{MV}(\Gamma, v) - \Gamma) \cdot F^{-1}(1-\Gamma) - (1-\theta^*(\Gamma))C_r(\Gamma) - \theta^*(\Gamma) \cdot C_p(1-\Gamma) + a^*_{MV}(\Gamma, v_{MV}) \cdot v_{MV}$$

Precisely the same calculations as found in the proof of Proposition 1 establish the increasing differences of $(\Gamma; W)$. As such, U_{MV} possesses monotone comparative statics in W, according to the specified partial order. ■

Corollary 1. *Let $v_{DV} < \hat{v}_{DV}$. Then $(\Gamma^{**}, \theta^{**})$ and a^{**}_{DV} are weakly larger under $\hat{v}_{DV}$ than under v_{DV}.*

Proof of Corollary 1. Because U_i has increasing differences in (Γ, v_i) – by way of $a^*_i(\Gamma, v_i)$ – Γ^{**} is weakly increasing in v_{DV}, as are $a^{**}_{DV} = a^*_{DV}(\Gamma^{**}, v_{DV})$ and $\theta^{**} = \theta^*(\Gamma^{**})$. ■

B Incorporating Uncertainty

B.1 Incomplete Information about the Location of the Median Valuation

In this subsection, we allow the v_i to be stochastic rather than deterministic to probe the robustness of the importance of the median voter (see Lemma 4). Specifically, we more explicitly consider platform competition between two candidates, asking whether the median voter – or an analogue to it – provides an indication of the policy selected under majoritarian politics. We sketch the result below, but it broadly hews to the "median of medians" result from Calvert (1985) (see also Duggan (2008)).

Suppose two office-motivated candidates compete with binding policy platforms over a continuum of voters i of mass one. Let $v_i \overset{iid}{\sim} F(\cdot), \forall i$. We continue to assume that competition occurs over the optimal size of the intervention, given the optimal type of intervention. Two additional assumptions streamline the exposition. First, limit consideration to active policy interventions, i.e., $P \neq 0$ and/or $R \neq 0$. Second, let the maximizers of the following be single-valued and given by $\overline{\Gamma}^{**}$ and $\underline{\Gamma}^{**}$, respectively:

$$U^1_{MV}(\Gamma) = W(\Gamma) + (1-\Gamma) \cdot (-F^{-1}(1-\Gamma)) - (1-\theta^*(\Gamma)) \cdot C_r(\Gamma) - \theta^*(\Gamma) \cdot C_p(1-\Gamma) + v_{MV},$$

$$U^0_{MV}(\Gamma) = W(\Gamma) + \Gamma \cdot F^{-1}(1-\Gamma) - (1-\theta^*(\Gamma)) \cdot C_r(\Gamma) - \theta^*(\Gamma) \cdot C_p(1-\Gamma).$$

Per Lemma 4, $\overline{\Gamma}^{**} > \underline{\Gamma}^{**}$, where $\overline{\Gamma}^{**}$ is the most-preferred level of compliance for *ex post* compliers, and $\underline{\Gamma}^{**}$ is the most-preferred level of compliance for *ex post* non-compliers. Compliance decisions are, of course, dependent upon the size of the intervention. As such, if $v_i \geq F^{-1}(1-\overline{\Gamma}^{**}) =: \tilde{v}$, i's most-preferred intervention is $\overline{\Gamma}^{**}$. If $v_i < F^{-1}(1-\overline{\Gamma}^{**}) < F^{-1}(1-\underline{\Gamma}^{**})$, i's most-preferred intervention is $\underline{\Gamma}^{**}$. Let $G(v) = \Pr(v_{MV} \geq v)$.[22] Then $G(\tilde{v})$ is the probability that $\geq 50\%$ of the population will prefer the intervention $\overline{\Gamma}^{**}$.

It follows that candidates choose between $\overline{\Gamma}^{**}$ and $\underline{\Gamma}^{**}$. We may normalize the office-holding benefit to 1. If the probability with which ties are broken in favor of each candidate is 1/2, and we suppose the probability that a candidate's opposition chooses $\overline{\Gamma}^{**}$ is given by q, then a candidate chooses $\overline{\Gamma}^{**}$ if

$$\begin{aligned}
EU(\overline{\Gamma}^{**}) &\geq EU(\underline{\Gamma}^{**}) \Leftrightarrow \\
G(\tilde{v})(q/2 + (1-q)) + (1-G(\tilde{v}))(q/2) &\geq G(\tilde{v})((1-q)/2) + (1-G(\tilde{v}))(q + (1-q)/2) \Leftrightarrow \\
q/2 + G(\tilde{v})(1-q) &\geq (1-q)/2 + (1-G(\tilde{v}))q \Leftrightarrow \\
G(\tilde{v}) &\geq 1/2.
\end{aligned}$$

The final line indicates that it is a dominant strategy for the candidates to converge to the preferred policy of the "estimated median" (Calvert 1985), which in this case refers to the median order statistic. With a continuum (or taking the limit of n draws as $n \uparrow \infty$), the median order statistic converges to the median of the parent distribution with variance tending to zero. If $\overline{\Gamma}^{**} \geq 1/2$, then both candidates converge in equilibrium to propose $(\overline{\Gamma}^{**}, \theta^*(\overline{\Gamma}^{**}))$ as their platform, otherwise $(\underline{\Gamma}^{**}, \theta^*(\underline{\Gamma}^{**}))$, where $\theta^*(\underline{\Gamma}^{**}) \leq \theta^*(\overline{\Gamma}^{**})$.

B.2 Allowing for Imperfect Enforcement

Let the probability of an individual who takes $a = 0$ being mistaken for having taken $a = 1$ ("type I error") be denoted α and the probability that an individual who took $a = 1$ being mistaken for having taken $a = 0$ ("type II error") be denoted β, with $\alpha, \beta < 1/2$. A member of the population i then chooses $a = 1$ if

[22] Where the identity of MV is determined after the v_i are realized and platforms proposed.

$(1-\beta)R - \beta \cdot P + v_i \geq \alpha \cdot R - (1-\alpha)P \Rightarrow v_i \geq -(1-\beta-\alpha)(P+R)$.

The proportion of the population complying is then $1 - F(-(1-\alpha-\beta)(P+R))$, still denoted Γ. Letting $X = P + R$, we have $X(\Gamma) = -F^{-1}(1-\Gamma)/(1-\alpha-\beta)$.

The proportion of the population paying a fine P is $(1-\alpha)(1-\Gamma) + \beta\Gamma = (1-\alpha) - (1-\alpha-\beta)\Gamma$, so the cost of fines is $C_p((1-\alpha) - (1-\alpha-\beta)\Gamma)$.

The proportion of the population receiving a subsidy R is $\alpha(1-\Gamma) + (1-\beta)\Gamma = \alpha + (1-\alpha-\beta)\Gamma$, so the cost of subsidies is $C_r(\alpha + (1-\alpha-\beta)\Gamma)$.

The social benefit and sum of valuations for those choosing $a = 1$ remain unchanged as functions of Γ.

The incremental return of using punishments instead of rewards

$$U_{SP}(\Gamma, p) - U_{SP}(\Gamma, r) = C_r(\alpha + (1-\alpha-\beta)\Gamma) - C_p((1-\alpha) - (1-\alpha-\beta)\Gamma).$$

Taking the derivative with respect to Γ, we have $(1-\alpha-\beta)(C_r' + C_p') > 0$, so we continue to have increasing differences in (Γ, θ).

Taking the derivative instead with respect to α, we have $(1-\Gamma)(C_r'+C_p')$, so we have increasing differences in (θ, α). If taken with respect to β, we have $-\Gamma(C_r' + C_p')$ and decreasing differences with respect to (θ, β).

Considering (Γ, α):

$$\begin{aligned} \frac{\partial^2}{\partial\Gamma\partial\alpha} U_{SP}(\Gamma, p) &= \frac{\partial}{\partial\Gamma} - (1-\Gamma)C_p'((1-\alpha) - (1-\alpha-\beta)\Gamma) \\ &= C_p' + (1-\alpha-\beta)(1-\Gamma)C_p'' \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{\partial^2}{\partial\Gamma\partial\alpha} U_{SP}(\Gamma, r) &= \frac{\partial}{\partial\Gamma}(1-\Gamma)C_r'(\alpha + (1-\alpha-\beta)\Gamma) \\ &= -C_r' + (1-\alpha-\beta)(1-\Gamma)C_r'' \end{aligned} \tag{9}$$

Considering (Γ, β):

$$\begin{aligned} \frac{\partial^2}{\partial\Gamma\partial\beta} U_{SP}(\Gamma, p) &= \frac{\partial}{\partial\Gamma}\Gamma C_p'((1-\alpha) - (1-\alpha-\beta)\Gamma) \\ &= C_p' - (1-\alpha-\beta)\Gamma C_p'' \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{\partial^2}{\partial\Gamma\partial\beta} U_{SP}(\Gamma, r) &= \frac{\partial}{\partial\Gamma} - \Gamma C_r'(\alpha + (1-\alpha-\beta)\Gamma) \\ &= -C_r' - (1-\alpha-\beta)\Gamma C_r'' \end{aligned} \tag{11}$$

If the cost functions convex in addition to increasing, then (8) and (11) are, respectively, unambiguously positive and negative. If (9) and (10) are, respectively, positive and negative, then we may conclude that the social planner's objective function possesses monotone comparative statics in α and $-\beta$. The proposition below follows as long as, for all $\Gamma \in (\Gamma_0, 1)$, both of the following conditions hold:

$$\frac{C_r''(\alpha + (1-\alpha-\beta)\Gamma)}{C_r'(\alpha + (1-\alpha-\beta)\Gamma)} \geq \frac{1}{(1-\alpha-\beta)(1-\Gamma)} \ , \tag{12}$$

$$\frac{C_p''((1-\alpha) - (1-\alpha-\beta)\Gamma)}{C_p'((1-\alpha) - (1-\alpha-\beta)\Gamma)} \geq \frac{1}{(1-\alpha-\beta)\Gamma} \ . \tag{13}$$

Proposition 6. *Letting α be the probability that an individual choosing $a = 0$ is mistakenly attributed the action of $a = 1$ and β be the probability that an individual choosing $a = 1$ is mistakenly attributed the action of $a = 0$, the social planner's objective function displays monotone comparative statics in α and $-\beta$ as long as $C_r(\cdot)$ and $C_p(\cdot)$ are sufficiently convex, i.e., satisfy* (12) *and* (13)*.*

Proof of Proposition 6. The proof follows immediately from Lemma 7 and the increasing differences in (Γ, θ), (θ, α), and (Γ, α), as well as $(\theta, -\beta)$, and $(\Gamma, -\beta)$. ■

As the probability α of incorrectly classifying a member of the population that chose $a = 0$ as having chose $a = 1$ increases, punishments become less costly to administer, all else equal. One direct effect is to make the use of punishments more attractive relative to the use of rewards. While cheaper punishments can encourage the social planner to allow greater non-compliance, the supposition of sufficient convexity ensures that costs rise too quickly as non-compliance increases, thus putting upward pressure on the optimal share choosing $a = 1$ *ex post*. The complementarity between using punishments and inducing a high share of the population to choose $a = 1$ *ex post* only reinforce the direct effects. Increases in α then lead to an increase in the optimal share of the population choosing $a = 1$ *ex post*, favoring the use of punishments. An analogous logic describes how increases in the probability of classifying an individual choosing $a = 1$ as one who took $a = 0$ lead to lower optimal levels of compliance and favor the use of rewards instead of punishments as the optimal type of policy.

C Accounting for Unaffected Subpopulations

The model has thus far set aside the possibility that the policy applies only to a "subpopulation of interest," with the population at-large not confronted with a choice between $a = 1$ and $a = 0$. This may take one of two forms: 1) it is not possible to distinguish the subpopulation of interest from the rest of the population for the sake of applying the policy, 2) it is possible to administer the policy only to the subpopulation of interest. The former receives attention first, and it requires only an informal discussion as an application of earlier results covers this case without difficulty.

When the subpopulation of interest is indistinguishable from the population at-large and the subpopulation not of interest tacitly chooses $a = 1$ (e.g., non-drivers not speeding), it is as though the distribution of valuations for the whole population is a first-order stochastic increase of the distribution of valuations of the subpopulation of interest (drivers). Invoking Proposition 1(b), this favors the use of disincentives. In the example of discouraging speeding, this formalizes the intuition that rewarding a substantial segment of the population (non-drivers) for not speeding (when they were at no risk of doing so anyway) would be incredibly inefficient. From the utilitarian's perspective, these conditions favor the use of fines and inducing a high share of the population choosing to drive safely (or not at all), and the fines would likely be even larger if majority-preference dictated the choice of policy.

In the context of copyright for artistic works, or patents for inventions, government wishes to encourage innovation, but it is unable to target the subpopulation of possible innovators, artistic or otherwise. This constitutes the presence of a large subpopulation that is not disposed to "comply" with the behavior government wishes to encourage. As such, the distribution of valuations of the population as a whole is less prone to choose $a = 1$ than the distribution of valuations within the subpopulation of interest. Referencing Proposition 1(b) again, these conditions lead a utilitarian to favor using rewards to spur innovation by a small share of the overall population. The implies the majority-preference would be for a smaller-than-optimal reward, which foots with oft-heard complaints from innovators across fields, namely, that the reward is insufficient compensation for their creative effort.

In many circumstances, however, it is easy to differentiate those in a subpopulation of interest from those who are not. For instance, a policymaker may wish to target an industry. It is usually straightforward to identify firms from individuals and, further, firms in a certain industry from firms in other industries. Those who do not own cars would not be penalized for failure to possess vehicle registration, and those without cropland would be ineligible for farm subsides. Call the portion of the population that would receive neither a reward nor a punishment under a given policy the "unaffected subpopulation." The "subpopulation of interest" still refers to the portion of the population to whom any incentive or disincentive would apply.

If enforcement is able to discriminate between the subpopulation of interest and the rest of the population, then the analyses of the utilitarian's optimal policy hold without further modification. The policymaker may ignore redistributive implications for subpopulations not directly affected by the policy, as she could with redistributive implications for individuals in the subpopulation affected by the policy. In the analysis of the popular support for incentive and disincentive policies, however, the presence of an unaffected subpopulation will materially affect the analysis. The unaffected subpopulation certainly reaps social benefit from compliance with the desired behavior in the subpopulation of interest. Furthermore, members of the unaffected subpopulation must also contribute to the financing of subsidies, but they may likewise benefit from the redistribution of fines or taxes collected.

Let the size of the subpopulation not directly affected by the policy, i.e., not eligible for a reward or punishment, be given by $\lambda \in (0,1)$. Denote an arbitrary member of this group by ℓ. The entire population is still of mass 1, so the size of the subpopulation of interest is of mass $1-\lambda$.[23]

Under a policy that involves the use of rewards (as well as potentially punishments), ℓ would have to contribute $(1-\lambda)\cdot R\cdot(1-F(-R-P))+C_r((1-\lambda)\cdot[1-F(-R-P)])$ to finance the subsidy, but receive no compensation for her behavior other than social benefit given by $W((1-\lambda)\cdot[1-F(-R-P)])$. With regards to policies applied to those choosing $a=1$, then, ℓ's utility function takes the same form as a member of the subpopulation of interest who chooses $a=0$. In contrast, under a punishment-based policy, ℓ will not receive any fine, but she will receive $(1-\lambda)\cdot P\cdot F(-R-P)-C_p((1-\lambda)\cdot F(-R-P))$ and $W((1-\lambda)\cdot[1-F(-R-P)])$. Thus, with regards to policies applied to those choosing $a=0$, ℓ shares the same utility function as a member of the subpopulation of interest who chooses $a=1$. Accordingly, maximizing ℓ's utility entails comparing the most-preferred punishment-based policy for a member of the sub-population of interest who *ex post* chooses $a=1$ to the most-preferred reward-based policy for a member of the sub-population of interest who *ex post* chooses $a=0$, and then comparing the best of those to the utility ℓ receives in the absence of any further policy intervention, namely $W((1-\lambda)(1-F(0)))$.

The result below characterizes the preferences of a member of an unaffected subpopulation, focusing on the utilitarian-optimal policy intervention and the majority-preferred policy intervention. Assume that the share of the population that is unaffected by the policy is given by $\lambda \in (\frac{1}{2},1)$, such that a majority of the population will receive neither incentive nor disincentive under a policy intervention.

Corollary 2. *When the unaffected subpopulation is larger than the subpopulation of interest, majority-preferred policy will entail either a larger-than-optimal disincentive applied to those choosing $a=0$ or a smaller-than-optimal incentive applied to those choosing $a=1$ (potentially a "negative reward," punishing those who choose $a=1$).*

The optimal policy displays monotone comparative statics with respect to pointwise increases in the marginal benefit to society of compliance.

Proof of Corollary 2. The arguments above establish that the majority-preferred policy intervention will be the more preferred of the complier-preferred disincentive and the non-complier-preferred incentive. Moreover, as the $(1-\lambda)$ drops out of the first-order condition, these policies coincide with the majority-preferred policies in the prior section. The comparative statics of the optimal policy intervention then follow immediately from Proposition 5. ■

[23]As above, it is never the case that $P \neq 0$ and $R \neq 0$.

When a subpopulation not directly affected by incentives or disincentives in a given policy domain is sufficiently large so as to decide the policy for the entire population, it will always be the case that rewards will achieve smaller than the socially optimal share of the population choosing $a = 1$ and punishments will achieve larger than the socially optimal share of the population choosing $a = 1$. This was the tendency noted in Proposition 4, but incorporating the unaffected subpopulation makes this a certainty.

An increase in the marginal benefit to society of members of the affected subpopulation choosing $a = 1$ rather than $a = 0$ increases the utility ℓ receives from P^{**} and decreases the utility ℓ receives from R^{**}. A similar statement holds for first-order stochastic increases in the distribution of valuations for choosing $a = 1$ rather than $a = 0$ among the subpopulation of interest. The downward pressure on rewards may drive them to become negative, constituting a disincentive for choosing $a = 1$.

The onerous fines that farms incur to receive organic certification are an example of a negative reward applied to those in a small subpopulation of interest that take a socially beneficial behavior. The low marginal benefit of a given (usually small) farm choosing to adopt organic practices favors the use of policies applied to those choosing $a = 1$, which further leads to smaller-than-optimal (even negative) incentives. That such a policy would generate revenue to be redistributed among the population-at-large would only help to overcome the loss of social benefit for a member of the unaffected subpopulation.