Policymaking Precision and Electoral Accountability

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Abstract

A voter who dislikes uncertainty will trade some ideological congruence in an elected official for precision in policymaking. This paper explores the mechanisms and consequences of voters holding politicians accountable based on their competence at executing the policies they intend to. Specifically, we consider two different moral hazards: the first over the policy selected, and the second over investment in increased precision. We find that the voter cannot induce an incumbent to choose a policy other than her ideal point in any equilibrium. The voter can, however, induce incumbents to put forward effort to improve their policymaking precision. In fact, and in contrast to other bases for accountability throughout the literature, policymaking precision provides a benefit to all risk-averse actors who care about policy. In this setting, then, it is no surprise to see term-limited incumbents still make investments in their competence. Less expected is that the possibility of being retained despite being a high-variance lawmaker exerts downward pressure on incumbents' first-period investments in policymaking precision.

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1 Introduction

Shepsle (1972) observed that the "act of voting, like gambling or purchasing insurance, is one involving 'risky' alternatives" (p. 560). When studying ways in which voters might overcome this risk, the electoral accountability literature has traditionally focused on two mechanisms. First, based on the spatial model of politics, scholars have examined whether voters can overcome adverse selection of politicians with respect to potentially dissimilar policy preferences. Second, much work has focused on voters' ability to select more competent politicians, where competence is a valence attribute orthogonal to policy preferences. We take a hybrid approach in this paper, engaging with both competence and spatial preferences.

While all voters should be concerned with the shared ideological congruence of candidates, risk-averse voters should consider also how precisely candidates are able to implement a given choice of policy. The model we present below incorporates the trade-off voters face between ideological congruence and a particular form of competence, namely, policymaking precision. Specifically, we ask whether and how voters might select those politicians who are able to implement their chosen policies more precisely.

Risk-aversion refers to preferences for greater certainty over outcomes, even at the expense of some expected quality of outcomes. Psychologically, risk averse individuals should be driven primarily by security rather than a desire for high returns (Schneider and Lopes 1986). In utility theory, risk aversion can be defined by preferring the expectation of a gamble to the gamble itself. Studies on risk attitudes across a variety of disciplines repeatedly shows that individuals are largely risk averse across a wide variety of behaviors. Holt and Laury (2002, 2005) conduct a series of experiments where participants are given a series of lottery choices for real cash payments and find that participants are largely risk averse even for small payments. As cash payments increase so does the level of risk aversion. Evidence of risk aversion is not confined to laboratory settings. Individuals have been shown to exhibit risk aversion regarding insurance markets (Binswanger 1980, Cohen & Einav 2007, Bosch-Domènech & Silvestre 1999), financial decisions (Nelson 2015), and game shows (Gertner 1993, Metrick 1995).

Research in political science echoes these findings. Prior work has shown voters to be predominantly, but not exclusively, risk averse and, further, shown that voters risk attitudes affect their vote choice. Risk-averse individuals are more likely to support incumbents over challengers (Morgenstern & Zechmeister 2001, Kam & Simas 2012), constituting a key determinant of the incumbency advantage (Eckles, Kam, Maestas & Schaffner 2014). Nadeau, Martin & Blais (1999) offers evidence that risk-averse voters give more weight to the perceived possibility of the worst case scenario while risk-loving voters engage in cost-benefit analysis when deciding among candidates. Tomz & Van Houweling (2009) finds that only risk-neutral or risk-loving voters do not punish candidates who take ambiguous policy positions. Individual's risk orientation also affects how they respond to policy frames, where those with higher levels of risk tolerance are more likely to select probabilistic policy outcomes over certain ones (Kam & Simas 2010). Bartels (1986) shows that voter's uncertainty of candidate positions has an effect similar to that of ideological distance on vote choice. The literature does not, however, display universal agreement over voters' level of risk aversion, with some empirical evidence suggesting that voters are not risk-averse in their voting behavior (Berinsky, Lewis et al. 2007).

Uncertainty appears in various ways across accountability models in spatial, rent-seeking, and multi-task environments. Previous work on electoral accountability features both moral hazard and incomplete (possibly asymmetric) information about politician quality. With both politicians' types and their actions not directly observable, voters have two sources of uncertainty to overcome. Politicians may vary in either their competence, willingness to shirk, or preferences, and voters want to select "good types." Politicians may also behave opportunistically and voters must sanction/reward behavior to limit shirking. Multi-task or pandering models, in the spirit of Holmstrom & Milgrom (1991), highlight the tension between expending effort towards activities that are objectively good for voters versus activities that are best for being re-elected (Ashworth 2005, Ashworth & Bueno de Mesquita 2006, Besley 2006, Canes-Wrone, Herron & Shotts 2001, Daley & Snowberg 2009).

Fearon (1999) focuses on the trade off between sanctioning and selecting politicians in a spatial setting. As long as voters believe there is any chance a politician is a good type, voters focus on selecting the politicians with similar preferences rather than sanctioning the policy outcome to induce implementation of the voter's ideal point. As voters become less informed about politicians' types, politicians are more likely to respond to electoral incentives, improving voter welfare (Besley 2006, Ashworth & Bueno de Mesquita 2014). In Banks & Sundaram (1998), politicians vary in their cost at providing public goods and lower types (higher cost) provide more public goods than higher-types in order to be re-elected.

Duggan & Martinelli (2017) show that in a two-period adverse selection selection model all politician types have electoral incentives to select the median ideal policy. This result generally holds in the infinite horizon setting (Duggan et al. 2000, Banks & Duggan 2008). Convergence depends crucially on the fact voters cannot perfectly observe politician type allowing extreme candidates to behave like moderate ones. Politicians only have incentives to represent voters if it is possible to obscure their own preferences for voters. If their true preferences are known with certainty than voters will select the politician with the closest ideal policy. Somewhat paradoxically to the folk theory of democracy, "asymmetric information can facilitate responsive democracy, where as full transparency leads to shirking" (Duggan & Martinelli 2017, 935).

We are motivated by the fact that policy-motivated and risk-averse voters will care about politician precision. Supporting this assumption, Yakovlev (2011) finds that greater risk aversion concerning the challenger enables incumbents to move policy away from the median voter while still being re-elected. Rational risk averse voters may prefer a precise yet ideologically incongruous politician over a congruent yet imprecise one. Previous work has considered politician competence in a rent-seeking environment where politician "type" is determined by their ability to produce valence outcomes or integrity (Berganza 2000, Fox & Shotts 2009). In the spatial setting, as noted, uncertainty was assumed to surround politicians' true preferences and not their ability to secure a desired outcome.

We contribute to the existing literature by bringing spatial politics into part of accountability literature that traditionally viewed competence as a valence charcteristic. This paper presents a spatial model where voters develop a retention rule on the basis of policy and competence, defined as policymaking precision that reduces the variance of policy outcomes for a given implemented policy. Specifically, the voter observes how close the realized policy hews to a particular policy, updates her beliefs about the incumbent's policy-making ability (i.e., precision), and retains the incumbent if and only if the realized policy is sufficiently close to the intended policy. For the voter's beliefs to be consistent in equilibrium, the incumbent must find it optimal to enact this given policy in equilibrium given the retention rule.

Reminiscent of Fearon's (1999) insight that selecting on competence interferes with voters' ability to enforce a given policy stance, we find that the only announced policy for which the voter can hold the politician accountable is the politician's ideal point. While voters in our model can not induce policy congruence, they are able to select politicians on the basis of competence and, in so doing, incentivize politicians to invest in competence. In contrast to rent-seeking environments, politicians prefer competent office-holders, even when they lose. This motivates politicians to invest in competence but also has a somewhat unexpected additional effect. Investing in competence increases the likelihood an incumbent is retained despite being a low-precision type, which exerts downward pressure on their incentive to invest in their own policymaking precision.

This paper proceeds as follows. After presenting the details of the model, we proceed to derive and analyze the equilibrium. We briefly explore an extension involving durable investments in policymaking precision before concluding with a discussion of directions in which the remarkably tractable model below may be taken.

2 Model Preliminaries

The model has three players: an incumbent (I), a challenger (C), and a voter (V). The game consists of two periods, $t = 1, 2$, each of which entail a politician $p \in \{I, C\}$ implementing a policy $x_{p,t} \in \mathbb{R}$. Given an implemented policy $x_{p,t}$, the realized policy is $y_{p,t} \sim \mathcal{N}(x_{p,t}, \tau_{p,t})$, where $\tau_{p,t}$ is the implementing politician's policymaking precision (i.e., the inverse of the variance).

In the first period, the incumbent implements a policy $x_{I,1} \in \mathbb{R}$. All players incur payoffs from the realized $y_{I,1}$, and the voter then decides to retain I or replace her with C. In the second period, the winning politician selects $x_{p,2}$, the players incur payoffs from the realized $y_{p,2}$, and the game ends.

All players have an ideal point $\hat{x}_i \in \mathbb{R}$, $i = I, C, V$ (imposing only $\hat{x}_I > \hat{x}_V$) and incur quadratic loss in the distance from their ideal points to realized policies. In addition to the policy component of their preferences, politicians also receive a benefit $\beta > 0$ from holding office. In each policymaking stage, the politician in office may invest $e_{p,t} \geq 0$ in her policymaking precision. The cost of effort is linear in the amount put forward, $e_{p,t}$. None of the players discount second-period payoffs.

The ideal points and preferences are all common knowledge. The voter (and every other player) observes only $y_{I,1}$ when making her retention decision, but neither the policy choice, $x_{I,1}$, nor the effort put forth, $e_{I,1}$. Each politician has a type, θ_p, which is constant across periods and which we often refer to as the politician's competence. All players know only that $\theta_p \sim Gamma(\alpha_p, \lambda_p)$, for each $p \in \{I, C\}$, where $\alpha > 1$ is the shape parameter and $\lambda > 0$ is the rate (inverse-scale parameter).[1] A politicians policymaking precision is given by $\tau_{p,t} = e_{p,t} \cdot \theta_p$. In this formulation, higher types are more competent in the sense that they make policy with greater precision or, equivalently, lower variance.

Throughout, we employ the solution concept of weak perfect Bayesian equilibrium (henceforth, "equilibrium"). In the context of accountability models, the requirement of consistency of beliefs ensures that unobserved policies $x_{p,t}$ and precision investments $e_{p,t}$ are nonetheless as good as known in equilibrium. For instance, consistency of beliefs and the fact that the gamma distribution is closed

[1] Ashworth & Bueno de Mesquita (2017, p. 1376) present a compelling argument in favor of assuming symmetric uncertainty around competence in accountability models, vis-à-vis policy preferences, for example, where an assumption of asymmetric information makes more sense.

under multiplication by scalars implies that $\tau_{p,t} \sim Gamma(\alpha_p, \lambda_p/e_{p,t})$.

It is reasonable to assume that neither politician's expected type is so poor that she would rather, *ex ante*, have her opponent hold office than herself. Assumption 1 invokes a condition to this effect. In fact, the condition in (1) is stronger than needed under the possibility of investments in precision.

Assumption 1. *If there were only a single-round of policymaking, each politician would rather hold office than have the opposing politician hold office, i.e.,*

$$\beta + (\hat{x}_I - \hat{x}_C)^2 \geq \left| \frac{\lambda_C}{\alpha_C - 1} - \frac{\lambda_I}{\alpha_I - 1} \right|. \tag{1}$$

Further, let the benefit to holding office and the candidates' ideal policies be such that $\beta + (\hat{x}_I - \hat{x}_C)^2 > 1$.

It is instructive to demonstrate why the condition in (1) embodies the substance of the assumption in the absence of possible investments in precision. Suppose there is no effort decision, $e_{p,t}$, letting $\tau_{p,t} = \theta_p$. We begin with a fundamental result from distribution theory.

Lemma 1. *If* $y|(x;\tau) \sim \mathcal{N}(x, \tau)$, *and* $\tau \sim Gamma(\alpha, \lambda)$, *then* $y|x \sim t_{2\alpha}(x, \lambda/\alpha)$, *i.e., a t-distribution with* 2α *degrees of freedom, location parameter* x, *and scale parameter* λ/α, *which has a mean of* x *and variance* $\lambda/(\alpha - 1)$.

All proofs may be found in Appendix A. Denote the density of a random variable y that is distributed according to a t-distribution with ν degrees of freedom, location parameter μ, and scale parameter σ by $p_\nu(y|\mu, \sigma)$ and its cumulative distribution function by $P_\nu(y|\mu, \sigma)$. Note that σ is not the standard deviation here, but rather a scale parameter for a non-standard t-distribution.

Note also that if y is a random variable with mean μ and variance Σ, then

$$\begin{aligned}
\mathbb{E}[(y - \hat{x})^2] =& \mathbb{E}[y^2] - 2\mathbb{E}[y]\hat{x} + \hat{x}^2 \\
=& \Sigma + \mu^2 - 2\mathbb{E}[y]\hat{x} + \hat{x}^2 \\
=& \Sigma + (\mu - \hat{x})^2.
\end{aligned}$$

If in office for a single period, candidates will optimally implement their own ideal point. The incumbent would then wish to be in office if

$$\begin{aligned} \beta - \mathbb{E}[((y|\hat{x}_I) - \hat{x}_I)^2] &\geq - \mathbb{E}[((y|\hat{x}_C) - \hat{x}_I)^2] \Rightarrow \\ \beta - (\hat{x}_I - \hat{x}_I)^2 - \lambda_I/(\alpha_I - 1) &\geq - (\hat{x}_I - \hat{x}_C)^2 - \lambda_C/(\alpha_C - 1) \Rightarrow \\ \beta + (\hat{x}_C - \hat{x}_I)^2 &\geq \lambda_I/(\alpha_I - 1) - \lambda_C/(\alpha_C - 1) \end{aligned}$$

Analogously, the challenger wishes to be in office if

$$\beta + (\hat{x}_C - \hat{x}_I)^2 \geq \lambda_C/(\alpha_C - 1) - \lambda_I/(\alpha_I - 1).$$

Combining the two inequalities, each candidate would rather hold office if

$$\beta + (\hat{x}_C - \hat{x}_I)^2 \geq \left| \lambda_C/(\alpha_C - 1) - \lambda_I/(\alpha_I - 1) \right|.$$

Assumption 1 will ensure the incumbent always seeks to be reelected. The logic will be similar to that underpinning the calculations above. In fact, re-incorporating the possibility of investments in precision will render Assumption 1 much stronger than necessary.

3 Equilibrium Analysis

Our analysis of the model proceeds as follows. We first characterize the challenger's optimal behavior in the second period, should she be in office. We then address the incumbent's optimal behavior, in the event that she is retained. Given second-period play, we may then determine the voter's retention rule, and given this retention rule, we are able to formulate the incumbent's first-period maximization problem. Finally, we address consistency of beliefs, ensuring that the voter's expectations of the incumbent's first-period behavior are correct in equilibrium.

A candidate setting policy in the second period will optimally implement her own ideal point. Since the gamma distribution is closed under multiplication by scalars, transforming as we noted above, we may apply Lemma 1 and note $y|(x,e) \sim t_{2\alpha}(x, \lambda/(\alpha e))$, then $\mathbb{E}(y|x,e) = x$, and $Var(y|x,e) = \frac{\lambda}{(\alpha-1)e}$. Knowing $y_{p,2}|(\hat{x}_p, e_{p,2}) \sim t_{2\alpha}(x, \lambda/(\alpha e_{p,2}))$, the second-period officeholder determines the optimal investment in policymaking precision by solving:

$$\begin{aligned}
\max_{e_{p,2}\in\mathbb{R}_+} -Var(y|\hat{x}_p, e_{p,2}) - e_{p,2} &\Rightarrow \\
\max_{e_{p,2}\in\mathbb{R}_+} -\frac{\lambda}{(\alpha-1)e_{p,2}} - e_{p,2} &\Rightarrow \\
e^*_{p,2} = \sqrt{\frac{\lambda}{\alpha-1}}&. \qquad (2)
\end{aligned}$$

The second-period officeholder's equilibrium policymaking variance is given by the same quantity, and her value function is $-2\sqrt{\lambda/(\alpha-1)}$. While there are no electoral benefits of costly investment in the final stage of the game, it is still of benefit to the second-period officeholder.

The voter then calculates a payoff of $-\sqrt{\lambda_C/(\alpha_C-1)} - (\hat{x}_C - \hat{x}_V)^2$ from electing the challenger. If the incumbent is in office in period 2, however, the voter will have observed a policymaking outcome in period 1 and updated her beliefs about the incumbent's type. Since both the incumbent and the voter shared the same uncertainty about the incumbent's type, the incumbent will update in precisely the same manner as the voter, in a manner specified by in the next result.

Lemma 2. *Given an observed $y_{I,1}$ realized from a choice of first-period policy $x_{I,1}$ and first-period investment $e_{I,1}$, $\theta_I|(y_{I,1}; x_{I,1}, e_{I,1}) \sim Gamma(\alpha'_I, \lambda'_I e_{I,1})$, where $\alpha'_I = \alpha_I + 1/2$ and $\lambda'_I = \lambda_I/e_{I,1} + (y_{I,1} - x_{I,1})^2/2$.*

Applying Lemmas 1 and 2, as well as the fact that the incumbent will optimally choose $x_{I,2} = \hat{x}_I$ and some level of effort $e_{I,2}$, the incumbent's second period policy has a density given by:

$$p_{2\alpha_I+1}\left(y|\hat{x}_I, \frac{\lambda_I + e_{I,1}(y_{I,1}-x_{I,1})^2/2}{e_{I,2}(\alpha_I+1/2)}\right).$$

The second-period policy realization thus has a mean of $\hat{x}_I$ and a variance of $\frac{\lambda_I + e_{I,1}(y_{I,1}-x_{I,1})^2/2}{e_{I,2}(\alpha_I-1/2)}$, the scale parameter multiplied by $\frac{\nu'}{\nu'-2}$, where $\nu' = 2(\alpha_I + 1/2)$. By the formula for the optimal second-period investment in precision in (2), it follows that:

$$e^*_{I,2} = \sqrt{\frac{\lambda_I + e_{I,1}(y_{I,1}-x_{I,1})^2/2}{\alpha_I - 1/2}},$$

again resulting in an equilibrium variance of the same quantity.

The voter then retains the incumbent if:

$$-\sqrt{\frac{\lambda_I + e_{I,1}(y_{I,1} - x_{I,1})^2/2}{\alpha_I - 1/2}} - (\hat{x}_I - \hat{x}_V)^2 \geq -\sqrt{\frac{\lambda_C}{\alpha_C - 1}} - (\hat{x}_C - \hat{x}_V)^2. \tag{3}$$

Labeling $\xi := 2(\hat{x}_I - \hat{x}_C)(\hat{x}_V - (\hat{x}_I + \hat{x}_C)/2)$ and $\kappa := \sqrt{\frac{\lambda_C}{\alpha_C - 1}}$, then the voter retains the incumbent as long as the observed $y_{I,1}$ satisfies:

$$y_{I,1} \lesseqgtr x_{I,1} \pm \sqrt{2\left[(\xi + \kappa)^2(\alpha_I - 1/2) - \lambda_I\right]/e_{I,1}}.$$

Let this specify $y^*(x_{I,1}, e_{I,1})$ and $y_*(x_{I,1}, e_{I,1})$, the bounds of a retention region that is symmetric about $x_{I,1}$. Note that the greater first-period effort, the more closely the policy realization must hew to the implemented policy for the voter to retain the incumbent. Investment inflates the precision of the incumbent, so the voter must deflate her inference.

The incumbent takes the retention region as given, i.e., not as a function of her first-period choices. Indeed, her first-period choices must be optimal given a retention region of $[\underline{y}, \overline{y}]$, which must in equilibrium equal $[y_*, y^*]$. The probability with which the incumbent believes she will be retained is thus $P_{2\alpha_I}(\overline{y}|x_{I,1}, \lambda_I/(\alpha_I e_{I,1})) - P_{2\alpha_I}(\underline{y}|x_{I,1}, \lambda_I/(\alpha_I e_{I,1}))$.

The incumbent then solves:

$$\begin{aligned}
&\max_{x_{I,1}, e_{I,1}} \; -(x_{I,1} - \hat{x}_I)^2 - \frac{\lambda_I}{(\alpha_I - 1)e_{I,1}} - e_{I,1} + \\
&\left[P_{2\alpha_I}(\overline{y}|x_{I,1}, \lambda_I/(\alpha_I e_{I,1})) - P_{2\alpha_I}(\underline{y}|x_{I,1}, \lambda_I/(\alpha_I e_{I,1}))\right] \cdot \\
&\left(\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda_C}{\alpha_C - 1}} + \mathbb{E}\left[-2\sqrt{\frac{\lambda_I + e_{I,1}(y_{I,1} - x_{I,1})^2/2}{\alpha_I - 1/2}} \,\middle|\, y_{I,1} \in [\underline{y}, \overline{y}]\right]\right).
\end{aligned}$$

The expectation term is the incumbent's second-period value function, adjusted for the fact that $y_{I,1}$, which enters into her updated beliefs about her latent type, was sufficiently close to $x_{I,1}$ for her to have been retained. Lemma 3 establishes a rather remarkable simplification of this expectation term, leading to the following, equivalent maximization problem:

$$\max_{x_{I,1},e_{I,1}} -(x_{I,1}-\hat{x}_I)^2 - \frac{\lambda_I}{(\alpha_I-1)e_{I,1}} - e_{I,1} +$$
$$\left[P_{2\alpha_I}(\overline{y}|x_{I,1},\lambda_I/(\alpha_I e_{I,1})) - P_{2\alpha_I}(\underline{y}|x_{I,1},\lambda_I/(\alpha_I e_{I,1}))\right] \cdot \left(\beta + (\hat{x}_I - \hat{x}_C)^2 + \sqrt{\frac{\lambda_C}{\alpha_C - 1}}\right) +$$
$$\left[P_{2\tilde{\alpha}_I}(\overline{y}|x_{I,1},\lambda_I/(\tilde{\alpha}_I e_{I,1})) - P_{2\tilde{\alpha}_I}(\underline{y}|x_{I,1},\lambda_I/(\tilde{\alpha}_I e_{I,1}))\right] \cdot \left(\sqrt{\lambda_I(\alpha_I - 1/2)}\left(\frac{\Gamma(\alpha_I - 1/2)}{\Gamma(\alpha_I)}\right)^2\right)$$

where $\tilde{\alpha} = \alpha - 1/2$.

The solution to this problem defines optimal $(x^*_{I,1}, e^*_{I,1})$ that are functions of the bounds of the retention region. The voter's problem implicitly defined optimal bounds for retention as a function of first-period policy location, $x_{I,1}$, and effort, $e_{I,1}$. Equilibrium then requires that these values are fixed points of the composed best-response functions.

Although we allow for arbitrary policies, in equilibrium the incumbent will propose her ideal point in both periods. Appendix B demonstrates this result by first considering the optimal choice of policy without an investment decision. From there, it is easy to show the same result obtains in the presence of a precision-investment decision, as well. Furthermore, as noted in the Introduction, Fearon (1999) establishes that the possibility of holding elected officials accountable precludes any residual enforcement of behavior that, known in expectation, does not affect inference about politician type.

The first-order condition corresponding to the maximization problem above compares straightforwardly, albeit equivocally, to single-period considerations. The first part of the objective function is simply the maximization problem that a politician limited to a single term would face. Our main result establishes that, while first-period investment will be greater if the incumbent were limited to just one term, the possibility of retention exerts both upward and downward pressure on the extent of this increased investment.

Proposition 1. *First-period incumbent investment with the possibility of retention will be higher than without the possibility of retention. The extent to which first-period investment is greater increases in the size of the office-holding benefit, the ideological distance between the incumbent and the challenger, and the expected variance of the challenger's policy realizations; it decreases in the expected variance of the incumbent's own policy realizations.*

The possibility of retention exerts upward pressure on the incumbent's first-period effort because it increases the probability of the policy realization lying in the retention region, and she seeks to be retained in order to reap the office-holding benefit, the ability to implement her own ideal policy rather than the challenger's, and to avoid the variance in policy realizations associated with the challenger. Yet the incumbent understands that her effort increases the chance that she will be retained despite being of low type, i.e., low policymaking precision. The incentive effect suffers from the benefit the incumbent derives from being of high type. The downward pressure on the incentive effect from the incumbent's desire to be of high type may be counter-intuitive for at least two reasons. First, effort by the incumbent that enhances reelection prospects in other accountability settings does not confer any cost other than the cost of the effort itself. In our setting, however, because the incumbent herself benefits from the same selection effects that the voter does, she prefers not to inflate the estimate of her quality simply to improve her chance of being reelected. Second, it may seem odd that a politician would reduce their own reelection prospects for the sake of being retained only if high enough quality. If we truly believe this lacks verisimilitude, then the implication is that β is quite large relative to policy motivations, in which case the incentive effect of reelection on effort would be strong.

4 Discussion

This paper presents a spatial model where voters hold politicians accountable for their competence at executing the policies they intend to. Voters, while knowledgeable about politicians' ideal points, are uncertain about their competence and use realized policies as the basis for evaluating policymaking precision relative to politicians' implemented policies. We find that voters use a retention rule that serves as a selection mechanism on the basis of politician competence, though the incumbent's own desire to be retained only when she is a high type tempers the effectiveness of this mechanism.

While politician errors might sometimes work in the voter's favor, the realized policy is just as likely to work against them. Voters therefore are willing to trade this policy loss for less erratic politicians. This result stands in stark contrast to the majority of electoral accountability that show politicians implementing the voter's ideal policy or some policy between the voter's and politician's ideal points. Electoral incentives may encourage politicians to put forward effort to improve their policymaking precision, and indeed politician's may put forth this sort of effort even without the possibility of retention. Other models feature the unrealistic prediction that term-limited politicians expend zero effort. That is not the case in this model, where investments in precision make all players in our model – the voters, elected politicians, and losing politicians – better off.

While at first glance the inability of voters to induce congruence suggests poor government accountability, policymaking precision is as important for government performance as the policies that incumbents promise. If voters are risk averse and care about policy outcomes, then they are best served implementing an accountability mechanism on the basis of policymaking precision. This result perhaps reconciles high incumbent reelection rates with increasingly polarized politicians.

Finally, it is worth noting that our model is fully compatible with citizen-candidate models of electoral competition. As with all accountability models, we assume politicians are citizen candidates in the sense that binding platforms are not possible. Further, we assume ideal points are known, as they would be in any citizen-candidate model – an assumption not common in accountability models with spatial preferences. Moreover, and seemingly unique to our model of accountability with spatial preferences, all politicians implement their ideal policy in all periods. Such behavior is the premise of citizen-candidate models of electoral competition. The citizen-candidate class of models has yielded a number of insights, especially around entry into elections. Our accountability model could be merged with such models to speak to both self-selection by politicians into campaigns and voters' selection of incumbents in subsequent elections.

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A Proofs of In-Text Results

Proof of Lemma 1. Recall the density of $y|(\tilde{x};\tau)$ is $\sqrt{\frac{\tau}{2\pi}}e^{-\tau(y-x)^2/2}$, and the density of θ is $\frac{\lambda^\alpha}{\Gamma(\alpha)}\tau^{\alpha-1}e^{-\lambda\tau}$. Integrating τ out, the density of $y|\tilde{x}$ is

$$\begin{aligned}
&\int_0^\infty \sqrt{\frac{\tau}{2\pi}}e^{-\tau(y-\tilde{x})^2/2}\frac{\lambda^\alpha}{\Gamma(\alpha)}\tau^{\alpha-1}e^{-\lambda\tau}d\tau \\
&\frac{1}{\sqrt{2\pi}}\frac{\lambda^\alpha}{\Gamma(\alpha)}\int_0^\infty \tau^{\alpha-1/2}e^{-\tau(\lambda+(y-\tilde{x})^2/2)}d\tau \\
&\frac{1}{\sqrt{2\pi}}\frac{\lambda^\alpha}{\Gamma(\alpha)}\frac{\Gamma(\alpha+1/2)}{(\lambda+(y-\tilde{x})^2/2)^{\alpha+1/2}} \\
&\frac{\Gamma(\alpha+1/2)}{\sqrt{2\pi\lambda}\Gamma(\alpha)}(1+(y-\tilde{x})^2/(2\lambda))^{\alpha+1/2} \\
&(\text{letting } \nu=2\alpha, \mu=\tilde{x}, \sigma^2=\lambda/\alpha) \\
&\frac{\Gamma((\nu+1)/2)}{\sqrt{2\alpha\sigma^2\pi}\Gamma(\nu/2)}(1+(y-\tilde{x})^2/(2\alpha\sigma^2))^{(\nu+1)/2} \\
&\frac{1}{\sqrt{\nu}\sigma}\frac{\Gamma((\nu+1)/2)}{\Gamma(1/2)\Gamma(\nu/2)}\left(1+\left(\frac{y-\tilde{x}}{\sqrt{\nu}\sigma}\right)^2\right)^{(\nu+1)/2}
\end{aligned}$$

This is the density of a random variable y distributed according to a t-distribution with 2α degrees of freedom, location parameter $\tilde{x}$, and scale parameter λ/α. It follows that $\mathbb{E}(y)=\tilde{x}$ and $\mathbb{E}(y-\tilde{x})^2 = \frac{\nu}{\nu-2}\sigma^2 = \frac{2\alpha}{2\alpha-2}\frac{\lambda}{\alpha} = \frac{\lambda}{\alpha-1}$.[2] ■

Proof of Lemma 2. The density of $y|(x_{I,1}, e_{I,1})$ is

$$\sqrt{\frac{\tau_{I,1}}{2\pi}}e^{-\tau_{I,1}(y-x_{I,1})^2/2}\frac{(\lambda_I/e_{I,1})^{\alpha_I}}{\Gamma(\alpha_I)}\tau_{I,1}^{\alpha_I-1}e^{-(\lambda_I/e_{I,1})\tau_{I,1}}.$$

The distribution of $\tau_{I,1}|y_{I,1}$ (where $y_{I,1}$ is a realization of $y|x_{I,1}$) is then

$$\propto \tau_{I,1}^{\alpha_I-1/2}e^{-\left(\lambda_I/e_{I,1}+(y_{I,1}-x_{I,1})^2/2\right)\tau_{I,1}},$$

from which we observe $\tau_{I,1}|y_{I,1}$ must be distributed $Gamma(\alpha_I+1/2, \lambda_I/e_{I,1}+(y_{I,1}-x_{I,1})^2/2)$. Since $\tau_{I,1}=\theta_I e_{I,1}$, $\theta_I|y_{I,1} \sim Gamma(\alpha_I+1/2, \lambda_I+e_{I,1}(y_{I,1}-x_{I,1})^2/2)$. ■

[2] ***Casella & Berger cite needed, or Severini, or Ferguson even, maybe.

Lemma 3. *For $q \in [0,1]$:*

$$
\begin{aligned}
&\mathbb{E}\left[-2\left(\frac{\lambda_I + e_{I,1}(y_{I,1}-x_{I,1})^2/2}{\alpha_I - 1/2}\right)^q \middle| y_{I,1} \in [\underline{y}, \overline{y}]\right] \cdot \\
&\left[P_{2\alpha_I}(\overline{y}|x_{I,1}, \lambda_I/(\alpha_I e_{I,1})) - P_{2\alpha_I}(\underline{y}|x_{I,1}, \lambda_I/(\alpha_I e_{I,1}))\right] \\
&= \left(\frac{\lambda}{\alpha - 1/2}\right)^q \frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)} \frac{\Gamma(\tilde{\alpha})}{\Gamma(\tilde{\alpha}+1/2)} \left[P_{2\tilde{\alpha}}(\overline{y}|\tilde{x}, \lambda/\tilde{\alpha}) - P_{2\tilde{\alpha}}(\underline{y}|\tilde{x}, \lambda/\tilde{\alpha})\right],
\end{aligned}
$$

where $\tilde{\alpha} = \alpha - q$.

For $q = 1$, $\frac{\lambda}{\alpha-1/2}\frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)}\frac{\Gamma(\alpha-1)}{\Gamma(\alpha-1/2)} = \frac{\lambda}{\alpha-1/2}\frac{\alpha-1/2}{\alpha-1} = \frac{\lambda}{\alpha-1}$.

For $q = \frac{1}{2}$, $\sqrt{\frac{\lambda}{\alpha-1/2}}\frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)}\frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)} = \sqrt{\frac{\lambda}{\alpha-1/2}}(\alpha-1/2)\frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)}\frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)}$
$= \sqrt{\lambda(\alpha-1/2)}\left(\frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)}\right)^2$

Proof of Lemma 3.

$$
\begin{aligned}
&\int_{\underline{y}}^{\overline{y}} \left(\frac{\lambda + (\tilde{y}-\tilde{x})/2}{\alpha - 1/2}\right)^q \int_0^\infty \sqrt{\frac{\theta}{2\pi}} e^{-\theta(\tilde{y}-\tilde{x})^2/2} \frac{\lambda^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\lambda\theta} d\theta d\tilde{y} \\
&\int_{\underline{y}}^{\overline{y}} \left(\frac{\lambda + (\tilde{y}-\tilde{x})/2}{\alpha - 1/2}\right)^q \frac{\lambda^\alpha}{\sqrt{2\pi}\Gamma(\alpha)} \Gamma(\alpha+1/2)(\lambda + (\tilde{y}-\tilde{x})^2/2)^{-(\alpha+1/2)} d\tilde{y} \\
&\int_{\underline{y}}^{\overline{y}} \left(\frac{1}{\alpha - 1/2}\right)^q \frac{\lambda^\alpha}{\sqrt{2\pi}\Gamma(\alpha)} \Gamma(\alpha+1/2)(\lambda + (\tilde{y}-\tilde{x})^2/2)^{-(\alpha+1/2-q)} d\tilde{y} \\
&\int_{\underline{y}}^{\overline{y}} \left(\frac{1}{\alpha - 1/2}\right)^q \frac{\lambda^\alpha \lambda^{-(\alpha+1/2-q)}}{\sqrt{2\pi}\Gamma(\alpha)} \Gamma(\alpha+1/2)(1 + (\tilde{y}-\tilde{x})^2/(2\lambda))^{-(\alpha+1/2-q)} d\tilde{y} \\
&\left(\frac{\lambda}{\alpha - 1/2}\right)^q \frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)} \int_{\underline{y}}^{\overline{y}} \frac{1}{\sqrt{2\pi\lambda}} (1 + (\tilde{y}-\tilde{x})^2/(2\lambda))^{-(\alpha+1/2-q)} d\tilde{y}
\end{aligned}
$$

Setting $\tilde{\alpha} = \alpha - q$ yields the desired result. The derivations for $q = 1$ and $q = 1/2$ follow from the simplifications using the definition of the Gamma function, viz., $\Gamma(z) = (z-1)\Gamma(z-1)$. ■

Proof of Proposition 1. Sketch:

1. Use Gautschi's inequality, whereby $\frac{1}{\alpha} < \left(\frac{\Gamma(\alpha-1/2)}{\Gamma(\alpha)}\right)^2 < \frac{1}{\alpha-1}$, to establish that the FOC of everything in the second two lines of the maximization is concave and thus contributing to the optimal level of effort, not detracting.

2. The rest follows immediately.

■

B Policy Location without Effort Investments

B.1 Complete Information Setting

Suppose the incumbent's precision is known to be θ_I and the challenger's to be θ_C, and further suppose that $x_{I,1}$ is observable. In the second period, whichever politician the voter selects can do no better than to implement her own ideal point. As such, the voter prefers retaining I to C if

$$\begin{aligned} \int_{-\infty}^{\infty} -(y_{I,2}-\hat{x}_V)^2\sqrt{\theta_I}\,\phi\left(\sqrt{\theta_I}(y_{I,2}-\hat{x}_I)\right)dy_{I,2} > \\ \int_{-\infty}^{\infty} -(y_{C,2}-\hat{x}_V)^2\sqrt{\theta_C}\,\phi\left(\sqrt{\theta_C}(y_{C,2}-\hat{x}_C)\right)dy_{C,2} \Rightarrow \\ -1/\theta_I-(\hat{x}_I-\hat{x}_V)^2 > -1/\theta_C-(\hat{x}_C-\hat{x}_V)^2. \end{aligned} \tag{4}$$

The above comparison uses the fact that the expectation of the squared difference between a random variable and a given point is equal to the variance of the random variable plus the squared difference between the mean of the random variable and the given point. This identity will prove to be useful repeatedly in the sections below.

Examining the inequality above, we may observe that if $|\hat{x}_I-\hat{x}_V| = |\hat{x}_C-\hat{x}_V|$, then the voter retains the incumbent if her policymaking precision is greater than the challenger's. If the precision of the challenger and the incumbent are equal, the voter selects the candidate whose ideal point is closest to her own. These conclusions are independent of first-period behavior, as the voter does not learn anything in the first period that would inform her forward-looking decision.

The only condition under which it would be possible for the voter's retention decision to induce a certain first-period behavior is if (4) holds with equality.In this case, the retention rule might specify that the incumbent is only elected if $x_{I,1}=\hat{x}_V$. The voter, indifferent between retaining the incumbent and not, would be willing to adhere to this rule. The incumbent will select $\hat{x}_V$ in the first period if

$$\begin{gathered} -1/\theta_I-(\hat{x}_I-\hat{x}_V)^2+\beta-1/\theta_I-(\hat{x}_I-\hat{x}_I)^2 > -1/\theta_I-(\hat{x}_I-\hat{x}_I)^2+-1/\theta_C-(\hat{x}_I-\hat{x}_C)^2 \\ \beta-(\hat{x}_C-\hat{x}_V)^2 > -(\hat{x}_I-\hat{x}_C)^2, \end{gathered}$$

where the second line follows from first via the equation capturing the voter's indifference. A sufficient condition for the above inequality to hold is that the challenger's ideal point is closer to the voter's than to the incumbent's. If the challenger and incumbent find their ideal points on either side of the voter's, as we would often assume to be the case in models of two-party competition, then this condition is satisfied. Given the voter's retention rule, the incumbent's optimal first-period policy would be $x^*_{I,1} = \hat{x}_V$.

This equilibrium, however, is neither unique nor robust. Under the strict assumptions that lead to voter indifference, all manner of retention rules might be supported in some equilibrium. The retention rule just discussed is simply the best for the voter. Without the strict assumptions underlying voter indifference – as would be the case, generically – the only equilibrium would entail the incumbent selecting her ideal policy in the first period.

Is the flimsiness of the voter's ability to induce desired behavior from the incumbent in the first period an artifact of the complete information assumption, or an intrinsic limitation of selecting politician's based on their policymaking precision? To gain purchase on this question, we restore the assumption of symmetric uncertainty about politician's precision. In the next section, we explore the moral hazard problem that results if the voter cannot observe the incumbent's policy choice. In the section after that, we consider politician effort and suppose the voter can observe the incumbent's action but not the effort she puts forth. To preview our results, we find that selecting for precision tempers the incentive effect of political accountability in both of contexts under study.

B.2 The Infeasibility of Policy Convergence

Turning to environments with incomplete information, much of the analysis proceeds as it did above. The main analytical complications surround the voter and the incumbent updating their beliefs about the incumbent's policymaking precision based only on the realized policy in the first period. A few results about the distributions involved greatly mitigate these complications. The following lemma addresses the distribution of realized policies in the first period as well as the updating of beliefs about incumbent precision based on the realized policy. Note that while implemented policies are not observed *per se*, they are known in equilibrium, so updating may be conditioned on knowledge of the implemented policy, $x_{I,1}$.

Lemma 4. *Let $\tilde{y}$ be the policy realized from implementing $\tilde{x}$. Then $\theta|\tilde{y} \sim Gamma(\alpha + 1/2, \lambda + (\tilde{y} - \tilde{x})^2/2)$.*

If the incumbent is retained, then having observed $y_{I,1}$ result from an implemented $x_{I,1}$, it follows from the Lemma above that $\mathbb{E}(y_{I,2}|x_{I,2}, y_{I,1}, x_{I,1}) = x_{I,2}$ and $Var(y_{I,2}|x_{I,2}, y_{I,1}, x_{I,1}) = (\lambda_I + (y_{I,1} - x_{I,1})^2/2)/(\alpha_I - 1/2)$.

In the second period, whichever politician holds office will implement her ideal policy. The difference in utility for the voter between electing the Incumbent and the Challenger, having observed y_I^1 result from some $x_{I,1}$, is thus given by

$$\begin{aligned} \Delta_V :=& \mathbb{E}[(y_{C,2} - \hat{x}_V)^2 | x_{C,2} = \hat{x}_C] - \mathbb{E}[(y_{I,2} - \hat{x}_V)^2 | x_{I,2} = \hat{x}_I, y_{I,1}, x_{I,1}] \\ =& \frac{\lambda_C}{\alpha_C - 1} + (\hat{x}_V - \hat{x}_C)^2 - \frac{\lambda_I + (y_{I,1} - x_{I,1})^2/2}{\alpha_I - 1/2} - (\hat{x}_I - \hat{x}_V)^2 \end{aligned} \tag{5}$$

Examining this expression, we may observe that it naturally creates a interval retention strategy, with the retention region for realized first-period policies centered on the implemented first-period policy, $x_{I,1}$. Setting $\Delta_V \geq 0$, such that the voter weakly prefers to retain the incumbent, we obtain

$$(\alpha_I - 1/2)\left[\frac{\lambda_C}{\alpha_C - 1} + (\hat{x}_V - \hat{x}_C)^2 - (\hat{x}_I - \hat{x}_V)^2\right] - \lambda_I \geq (y_{I,1} - x_{I,1})^2/2.$$

This condition implies that if the realized $y_{I,1}$ is not too far from the implemented $x_{I,1}$ in either direction, then the voter would rather retain the incumbent than replace her with the challenger. Given the exogenous λ_p, α_p, and $\hat{x}_i$ parameters, the optimal region $[\underline{y}^*, \overline{y}^*]$ is a function of the endogenously-determined policy implemented by the incumbent in the first-period, $x_{I,1}$. Indeed, we may rewrite the optimal retention region in terms of only the upper bound, $[2x_{I,1} - \overline{y}^*, \overline{y}^*]$, and consider $\overline{y}^*$ to be the voter's best-response.

To develop the intuition, if $\lambda_C = \lambda_I = \lambda$, $\alpha_C = \alpha_I = \alpha$, and $|\hat{x}_I - \hat{x}_V| = |\hat{x}_V - \hat{x}_C|$, then the voter retains the incumbent if $y_{I,1}$ satisfies

$$(y_I^1 - x_{I,1})^2 \leq \lambda/(\alpha - 1).$$

This specifies a retention region $[\underline{y}, \overline{y}]$, where $\overline{y} = x_{I,1} + \sqrt{\lambda/(\alpha - 1)}$ and $\underline{y} = x_{I,1} - \sqrt{\lambda/(\alpha - 1)}$, which ensures that the incumbent's expected policymaking precision under the posterior beliefs is higher than under the prior beliefs.

One aspect of symmetric retention region may be somewhat counter-intuitive, namely, that the voter might throw out an incumbent whose realized policy erred (albeit quite substantially) in the direction of the voter's ideal point. Yet regardless of the direction, an errant first-period policy suggests low policymaking precision. Heuristically, it would not be incentive compatible for a risk-averse voter to retain such a politician because they are just as likely to err in the opposite direction, a potential loss that outweighs the possibility of another instance of accidental convergence.

We proceed next to analyze the incumbent's optimal policy choice in the first period, given an interval retention strategy $[\underline{y}, \overline{y}]$. It is helpful, however, to first build up the components of the incumbent's utility function before laying out the full maximization problem she solves. In the first period, the incumbent receives utility $\beta - \frac{\lambda_I}{\alpha_I - 1} - (\hat{x}_I - x_{I,1})^2$. She is reelected with probability $\Pr(y_{I,1} \in [\underline{y}, \overline{y}] | x_{I,1}) = P_{2\alpha_I}(\overline{y}; x_{I,1}, \lambda_I/\alpha_I) - P_{2\alpha_I}(\underline{y}; x_{I,1}, \lambda_I/\alpha_I)$, where $P(\cdot)$ is the cdf of the generalized t-distribution with 2α degrees of freedom, location parameter $x_{I,1}$, and scale parameter λ_I/α_I. If she is not retained, the incumbent receives utility of $-\frac{\lambda_C}{\alpha_C - 1} - (\hat{x}_I - \hat{x}_C)^2$ under the challenger. If the incumbent is retained, she knows her realized policy $y_{I,1}$ must have fallen in the retention region. More generally, the incumbent knows that she (as well as the voter) will have updated about her type, and it is this updated belief about her policymaking precision that she must account for when anticipating second-period utility. Her anticipated variance in the second round, which is a function of the first-period policy realization, must reflect the fact that the first-period policy realization was close enough to $x_{I,1}$ to be retained. The incumbent's second-period utility if retained is thus

$$
\begin{aligned}
&\beta - \int_{\underline{y}}^{\overline{y}} \frac{\lambda_I + (y_{I,1} - x_{I,1})^2/2}{\alpha_I - 1/2} p(y_{I,1}; x_{I,1}, \lambda_I/\alpha_I | y_{I,1} \in [\underline{y}, \overline{y}]) dy - (\hat{x}_I - \hat{x}_I)^2 \\
=&\beta - \frac{1}{\alpha_I - 1/2} \left(\lambda_I + \int_{\underline{y}}^{\overline{y}} (y_{I,1} - x_{I,1})^2/2 p(y_{I,1}; x_{I,1}, \lambda_I/\alpha_I | y_{I,1} \in [\underline{y}, \overline{y}]) \right) dy_{I,1} \\
=&\beta - \frac{1}{\alpha_I - 1/2} \left(\lambda_I + \frac{1}{2} \mathrm{Var}(y_{I,1} | x_{I,1}, y_{I,1} \in [\underline{y}, \overline{y}]) \right)
\end{aligned}
$$

If retained, the incumbent's expected second-period utility entails the variance of the truncated t-distribution according to which the first-period policy is distributed. Separate from her personal reelection concerns, the incumbent wishes to be as low variance (high type) as possible if reelected. Lemma 5 notes that this expression, multiplied by the probability with which the incumbent is retained, is minimized at $x_{I,1} = (\overline{y} + \underline{y})/2$.

Lemma 5. *Fix bounds $\underline{y} < \overline{y}$ where $\overline{y} - \underline{y} = Y$, and consider mean ($\mu$) shifts of a random variable y with a distribution F and a density f that is symmetric around a single mode.*

$$
(\overline{y} + \underline{y})/2 = \arg\min_{\mu} \int_{\underline{y}}^{\overline{y}} ((y - \mu)^2/2) f(y; \mu) dy
$$

The incumbent solves (omitting terms that are constant in $x_{I,1}$ for parsimony):

$$
\begin{aligned}
&\max_{x_{I,1}} -(\hat{x}_I - x_{I,1})^2 \\
&\qquad + \Pr(y_{I,1} \in [\underline{y}, \overline{y}]|x_{I,1}) \left[\frac{\lambda_C}{\alpha_C - 1} + (\hat{x}_I - \hat{x}_C)^2 + \beta - \frac{\left(\lambda_I + \frac{1}{2}\mathrm{Var}(y_{I,1}|x_{I,1}, y_{I,1} \in [\underline{y}, \overline{y}])\right)}{\alpha_I - 1/2} \right] \\
&= \max_{x_{I,1}} -(\hat{x}_I - x_{I,1})^2 + \Pr(y_{I,1} \in [\underline{y}, \overline{y}]|x_{I,1}) \left[\frac{\lambda_C}{\alpha_C - 1} + (\hat{x}_I - \hat{x}_C)^2 + \beta - \frac{\lambda_I}{\alpha_I - 1/2} \right] \\
&\qquad - \frac{1}{\alpha_I - 1/2} \int_{\underline{y}}^{\overline{y}} ((y_{I,1} - x_{I,1})^2/2) p_{2\alpha}(y_{I,1}|x_{I,1}, \lambda_I/\alpha_I) dy_{I,1}
\end{aligned}
\tag{6}
$$

With the exception of the first term, $-(\hat{x}_I - x_{I,1})^2$, the rest of the incumbent's objective function is maximized at $x_{I,1} = (\overline{y} + \underline{y})/2$. The first term is maximized at $x_{I,1} = \hat{x}_I$, as this represents the incumbent's desire to implement her ideal point in all periods. The next lemma formalizes that the solution must lie in between these two points.

Lemma 6. *Let the incumbent's optimal first-period policy be denoted $x^*_{I,1}$. If given a retention region, $[\underline{y}, \overline{y}]$, whose mid-point lies to the left of $\hat{x}_I$ (i.e., in the direction of $\hat{x}_V$), then $x^*_{I,1} \in ((\overline{y} + \underline{y})/2, \hat{x}_{I,1})$.*

The voter's retention rule supposes that it lies symmetrically around the incumbent's implemented policy. In equilibrium, her beliefs must be correct. The incumbent's implemented policy will always shade toward her ideal point and away from the mid-point of the retention region, however, creating a tension in equilibrium. The next proposition states that the only possible resolution to this tension is for the incumbent to implement $x_{I,1} = \hat{x}_I$ and the voter to employ a retention region centered on $\hat{x}_I$.

Proposition 2. *No equilibrium exists in which the incumbent's optimal first-period policy is not equal to her ideal point, $\hat{x}_I$. An equilibrium does exist in which $x^*_{I,1} = \hat{x}_I$ and in which the voter's retains the incumbent if*

$$
y_{I,1} \lesseqgtr \hat{x}_I \pm \sqrt{2(\alpha_I - 1/2)\left[\frac{\lambda_C}{\alpha_C - 1} + (\hat{x}_V - \hat{x}_C)^2 - (\hat{x}_I - \hat{x}_V)^2\right] - \lambda_I}\,.
$$

The first statement of the proposition stands in contrast to many accountability models. It is usually possible to construct an equilibrium in which the incumbent takes some action, which is costly to the incumbent but beneficial to the voter, in order to improve her reelection chances. Referred to as the incentive effect of accountability mechanisms, we find that this is shut down in an environment of asymmetric information about which policy is implemented and in which policymaking precision is the basis for retention.

The selection effect of accountability mechanisms is still present, however. The voter is able to retrospect and learn about the incumbent's type in order to make a more informed prospective decision. The bounds of the voter's retention region move closer to $\hat{x}_I$ the higher the mean of the prior distribution for incumbent type (α_I/λ_I), the smaller the challenger's anticipated precision, and the closer her ideal point is to the challenger's relative to the incumbent's.