The Process and Perils of Coming Around: The Assimilation of Political Appointees into Bureaucratic Agencies*

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March 12, 2024

Abstract

The tendency for political appointees to assimilate into the bureaucratic agencies that they lead is a recurring source of tension between appointees and the executives who appoint them. This paper employs a formal model to explore how appointees come around to the views of the civil servants whom they oversee. We conceptualize a bureaucrat as providing a cheap-talk message about privately-known, policy-relevant conditions to an appointee who uses that information to update her beliefs and set two types of policy. Though the bureaucrat's and appointee's preferences are aligned conditional on beliefs, the appointee's prior beliefs about the likelihood of various states of the world differ from the bureaucrat's. In equilibrium, truthful reporting and inducing belief convergence may be at odds, and we identify when the bureaucrat will strategically choose to issue false reports. We apply the model's insights to the budget process and to agency recommendations during the COVID-19 pandemic.

Keywords: Appointee assimilation, belief convergence, bureaucracy, cheap talk, non-common priors

Word count: 9,600

*The authors wish to thank seminar audiences at the 2023 SPSA meeting, the 2023 MPSA meeting, and the University of Rochester, with special thanks to Tasos Kalandrakis, Zhaotian Luo, and Kim Seoyeon.

1 Introduction

Within a political system with constant changes in executive oversight, career civil servants (bureaucrats) serve an essential role in ensuring a degree of institutional continuity in the agencies at which they work. Political appointees at the helm of a government agency depend on that agency's career civil servants for policy-relevant information. In turn, bureaucrats rely upon appointees' power to actuate policies. To achieve policy outcomes, appointees and bureaucrats require each other's cooperation.

Meanwhile, executives look to political appointees to supervise the work of bureaucrats in the course of managing a given agency or department and, crucially, to promote the administration's goals through the agency's decision-making processes. As such, executives often select appointees whose policy-relevant beliefs differ from the bureaucrats they will oversee. Differences in beliefs may introduce an adversarial element into the initial relationship between bureaucrats and appointees, even if the two have the same preferences conditional on beliefs. Since at least the second presidential administration of the United States, however, executives' relationships with their appointees have been fraught. Among the reasons for the tension between President and Cabinet is the tendency for political appointees to assimilate into the bureaucracy over time.[1]

Cronin (1979) offers an analysis of the Carter administration, as well as illustrations from a number of other presidencies, demonstrating that the tension between civil servant and appointee resolves over time. As one scholar of the American presidency writes, "...the same political appointees who are so suspicious at the beginning of an administration experience a "cycle of accommodation" with [careerists] and gradually come to appreciate their essential contribution to managing the government" (Pfiffner 1988, p. 90). Indeed, the conflict between appointees and bureaucrats often gives way to a new tension. Rather than relying on the appointee to exert influence on the bureaucracy on their behalf, executives ultimately fear appointees are exerting the bureaucracy's influence on them.

Despite the regularity with which such assimilation occurs, the mechanisms by which appointees come around—and, especially, the consequences for policymaking—remain opaque. In this paper, we demonstrate how appointee assimilation may arise from the codependent, albeit often adversarial, relationship between political appointees and civil servants. Specifically, we develop a model that examines bureaucrats' direct impact on the belief formation of political appointees and the potential

[1] Historically, the appointee was said (in rather outdated, colonial terms) to have "gone native" (Cronin 1979). Our first contribution is to suggest alternative terminology for this phenomenon, namely, "coming around." In addition to providing a much-needed update of terminology, it also suggests the process by which the appointee's beliefs change likely shares properties with the processes by which other political actors come around to new perspectives and preferences.

motivation that bureaucrats may have to strategically misreport information.

The bureaucrat learns the true state of the world at the start of the game and reports on the state, truthfully or not, to the appointee. With this report in-hand, the appointee must set proactive policy for the agency and, concurrently, react to emergent incidents. Proactive policies represent structural or institutional decisions. Reactive policies represent decisions about specific events. Both actors seek reactive policy that matches the true state of the world (regardless of the underlying probability of that state occurring) and proactive policy that reflects the likelihood of each state of the world (regardless of the current realized state of the world). The bureaucrat alone knows the true likelihood of each state. The appointee has her own beliefs about the likelihood of each state, but she is willing to update her beliefs and base (in a sense to be made precise below) her decision-making on the bureaucrat's report to some extent.

Examples of proactive vs. reactive policy may include setting structural monetary policy vs. responding to economic changes with stabilizing monetary policy; setting the focus and staffing at the Centers for Disease Control and Prevention vs. responding to warnings of emergent viral threats; and deciding police funding vs. reacting to a particular incident of police misconduct. In each of these examples, an appointee will use a bureaucrat's advice about handling the more pressing issue to inform forward-looking policy, as well. At times, however, a civil servant's structural and incidental goals may be at odds. For instance, perhaps a bureaucrat wishes to convince a skeptical appointee of the importance of climate change and has the opportunity to do so via a report on weather conditions in the western United States. Suppose that average temperatures are rising in those states but that the current period has shown a slight abatement. An accurate assessment of current conditions would be essential for responding to a wildfire that could break out. Yet, an accurate assessment of current conditions may not convince the appointee of the veracity of climate change and the need to set forward-looking policy in accordance.

We say the appointee has "come around" when the appointee's beliefs update in the direction of the bureaucrat's. Whether and to what extent the appointee comes around depends on the bureaucrat's equilibrium probability of accurately reporting each state of the world, with consequences for the appointee's proactive and reactive policy decisions. If the appointee's beliefs update such that they align more closely with the bureaucrat's, the bureaucrat can expect the appointee to enact proactive policies more in line with what the bureaucrat believes policy ought to be. If the appointee comes around in response to misinformation, the bureaucrat will have traded a more favorable proactive policy decision for a worse reactive policy decision.

The reliability of the information exchange depends on the initial similarity of bureaucrat and appointee beliefs as well as the importance placed on accurate reactive policies relative to proactive policies. When the bureaucrat and the appointee do not disagree to much about the underlying likelihood of each state, the bureaucrat truthfully reports the state of the world in equilibrium, and the appointee will come around precisely when she would have if she could observe the state of the world herself. When there is greater conflict between the views of the bureaucrat and the appointee, however, or when bureaucrats place less weight on appropriate reactive policies, the bureaucrat may engage in deception by reporting misinformation to the appointee in order to influence belief formation so that the political appointee comes around in cases when she would not have if she could observe the state of the world. In cases of extreme *ex-ante* disagreement, the bureaucrat issues a single report regardless of the state and the appointee does not come around at all, even in cases when she would have if she were able to observe directly the state of the world.

This phenomenon of belief convergence mirrors scenarios in which leaders allow new information or insights to shape their decisions. Indeed, the model could apply to executive oversight of the bureaucracy more broadly or, even more generally, to any sender-receiver relationship that satisfies a few basic assumptions. Specifically, the model presented herein has relevant implications for an advisee-advisor interaction as long as the advisee, facing two decisions, uses any (cheap-talk) message from the advisor about policy-relevant conditions to inform both decisions. We situate our model more precisely within cheap-talk and related theoretical literatures in the next section.

After the literature review, we proceed to lay out the details of the model, followed by our analysis. We then apply our insights gleaned from the model to better understand the dynamics within the executive branch of budgeting, including the peculiar case of deregulation in the 1980s. We also discuss the formulation of Coronavirus disease safety policies in light of the model's conclusions, asking why high-ranking political officials and the public were misled by elements within the bureaucracy at several points during the pandemic.

2 Literature Review

A number of complementary accounts—formal and otherwise— of coming around exist within the bureaucracy literature and in the economics literatures on cheap talk and belief convergence. Drawing on interviews of political appointees and career civil servants, Heclo (1977) argues that while appointees often mistrust the bureaucracy when first appointed, they can help themselves in pursuit of their goals by creating and reinforcing relationships of trust and confidence with their superiors and interest groups outside of an agency. He argues, however, that generally this "self-help" is not enough; in order for appointees to accomplish their goals and be effective leaders they must elicit the cooperation of subordinate civil servants to provide "knowledgeable continuity to warn and propose, and institutionalized responsiveness to help carry out executive decisions" (ibid., p. 170-171).

Since political appointees are typically at an informational disadvantage when they are appointed to leadership roles within an agency, given their lack of substantive expertise and familiarity with issues that arise within bureaucratic politics (see also Pfiffner (1988), pp. 93, 100), a primary responsibility of many bureaucrats within the bureaucracy is to explain the merits of policies to their appointee supervisors (Dolan 2000). Whatever control bureaucrats have over policy, it is often through the channel of educating and informing their inexperienced appointees, who then make the decisions about policy, a phenomenon present also outside of the American context (Berman, Chen, Jan & Huang 2013). Studies acknowledge that bureaucrats often hold a public service motivation that lasts over time and across regimes (Chen & Hsieh 2015, Miller & Whitford 2016), yet others recognize that bureaucrats desire more influence over policy outcomes than they perceive themselves as having relative to appointees (Stehr 1997).

That bureaucrats may be called upon for their expertise while simultaneously possessing policy agendas is well-trodden territory across the literature on the American bureaucracy (Peters 1981), including the subset of this literature using formal models to better understand executive and inter-branch relations. Most prior work on the strategic use of bureaucratic expertise, and the political trade-offs entailed in doing so, focuses on the principal-agent problems facing executives (Bendor & Meirowitz 2004, Epstein & O'Halloran 1999, Gailmard & Patty 2013, Gailmard 2022*a*, Gailmard 2022*b*, Huber & Shipan 2002) or Congress (Alesina & Tabellini 2007, Fox & Jordan 2011) when delegating to the civil service. Indeed, much of this work takes the civil service to be policymakers (see also Fox & Van Weelden (2012) and Ashworth & Sasso (2019)), ignoring the political intermediaries who manage each agency and whose direction and sign-off are required at least on high-level policy formulation in the agency if not also street-level implementation.

An exception is Warren (2012), which features a complete array of actors: Congress, civil servant (agent), appointee (supervisor), and executive, though the focus is primarily on the appointments and oversight functions. In this model, the appointee behaves as though she has come around, but Warren's account is based on differing preferences, and any appointee assimilation is, as the author notes, just policy compromise for the sake of inducing cooperation from the agent. In our model, the preferences of the appointee and bureaucrat are identical *conditional* on beliefs about the likelihood of each state of the world and the current state; it is their beliefs that differ. We return to the significance of these assumptions in the subsection "Comments on the Model's features."

The bureaucrat's report to the appointee in our model falls squarely within the definition of cheap talk (Crawford & Sobel 1982). The costless message, while potentially influential in equilibrium, does not directly affect the payoffs of either the bureaucrat or the appointee. With regards to the verifiability of the information from the bureaucrat, Gailmard & Patty (2012) lay out a "credibility continuum," in which bureaucrats as agents may provide messages that range from cheap talk to costly signals to fully verifiable (but with the option to issue a report or not). In our setting, the bureaucrat must issue a report (unlike Gailmard & Patty (2013), in which information may be withheld entirely), but the report need not be factual as it cannot be verified before the appointee acts.[2] The conclusions we draw about the nature of information transformation and subsequent changes in beliefs are reminiscent of the insights from the older, more abstract, literature on the "expert problem" in which a decision maker rationally updates her beliefs about the likelihood of some event of interest occurring in light of an expert's opinion (Morris 1974, French 1980, Genest & Schervish 1985).

As noted above, a key feature of our model is that the bureaucrat's message informs two simultaneous policy decisions facing the appointee. Our model thus has a strong resemblance to Farrell & Gibbons (1989), who consider the effect of constraining a sender playing two cheap-talk games to a single message that is observed by both receivers. They show it is possible to achieve communication in the bundled game when no (or only limited) communication was possible under two privately-observed messages. In contrast to this possible "disciplining" effect, they also demonstrate the potential for "subversion," in which the conflict in one game spoils the communication equilibrium in the other when the sender can only send one message. While our game differs in ways we make clear immediately below, the notions of discipline and subversion are quite useful in clarifying the structure and equilibria of our model.

The two decisions taken by the appointee are not both—taken in isolation—cheap-talk games. Since the appointee and bureaucrat both seek reactive policy that aligns with the state of the world, this aspect of the game does constitute a cheap-talk setting, albeit one devoid of any conflict. The players'

[2]See Denisenko, Hafer & Landa (2023a, 2023b), as well as models of Bayesian persuasion, for examples in which the sender's report must accurately reflect the sender's private information.

preferences around proactive policy decisions, however, do not depend on the state of the world but rather on (their beliefs about) the underlying likelihood of different states. As such, this element of the game is not, by itself, a cheap-talk game. Moreover, the payoffs from the proactive policy decision in our model are endogenous to the strategies employed in equilibrium, since the strategies determine the updated beliefs and thus the appointee's optimal proactive policy. This complication is not present in Farrell & Gibbons (1989). Nevertheless, the proactive policy decision takes on the flavor of cheap talk when yoked together with the reactive policy decision, and our analysis uncovers when the reactive policy decision disciplines communication around the proactive policy decision and when the latter subverts communication regarding the former.

Finally, while Farrell & Gibbons (1989) study a setting with two distinct audiences, a central and substantively important assumption of our model is that the same actor makes both decisions.[3] Even if the bureaucrat could issue two messages, the appointee could not credibly commit to using each message to inform just one decision. The assumption that the bureaucrat sends a single message in our model is less about the inability of the bureaucrat to send multiple messages and much more about the fact that a policymaker will use any piece of information she receives to inform all choices for which that information might be relevant.

Morris (2001) presents a model in which a receiver learns about the type of sender, and reputation concerns affect the sender's incentives to communicate truthfully. The model is akin to a version of Farrell & Gibbons (1989) with two private cheap-talk messages (where the two cheap-talk games are the hypothetical games with the two types of senders) with only a single policy decision.[4] In contrast, we focus on public communication with two policy decisions. More importantly, receptivity to the sender's message in Morris (2001) increases as a result of a more favorable impression of the sender's exogenously determined trustworthiness. In our model, the receiver's degree of trust in the sender's message emerges endogenously, and increasing receptiveness to the sender's message reflects the sender's reduced incentives to misreport in equilibrium.

In Morris (2001), as in Warren (2012), equilibrium behavior changes but without any accompanying underlying change in policy-relevant beliefs. We readily concede that reputation concerns or the need to ensure bureaucratic buy-in with policy decisions are consistent with and also explain some observed appointee responsiveness. Yet we see evidence-based belief change as a central factor in descriptions

[3]Farrell & Gibbons (1989) would describe well any scenario in which a bureaucrat advises multiple appointees, or perhaps an appointee and a staff member in the Office of the President, and in which the bureaucrat has the choice of issuing a private message to each or a more public message that both receivers observe.

[4]Unlike both our model and Farrell & Gibbons (1989), Morris (2001) models repeated interaction across two periods.

of appointee assimilation into bureaucratic agencies, some of which we discuss in the "Comments on the Model's Features" subsection below and several of which we have already mentioned.

In light of this mechanism, the work most similar to ours is Hirsch (2016), which is, to our knowledge, one of the few papers on the bureaucracy that also employs non-common priors among the key actors. Hirsch (2016) models a situation in which the appointee seeks to affect the beliefs of the agent, the opposite of our setting. Furthermore, the belief manipulation in his paper arises from experimentation (Bayesian persuasion). As noted, we do not impose a requirement for truthful reporting, as is fundamental in the Bayesian persuasion literature. Indeed, a central goal of our paper is to understand when civil servants will systematically misreport to their superiors. Nonetheless, our work shares the premise of appointees and bureaucrats having common goals but different beliefs.

We address our emphasis on belief rather than preference convergence in the "Comments on the Model's Features" subsection below, but we highlight here the literature that discusses belief convergence among players with non-common priors. Whether the players' differing prior beliefs need to have originated, pre-play, from a commmon set of beliefs has been the subject of much debate (Aumann 1976, Fey & Ramsay 2006, Gul 1998). Regardless of how they arose, Acemoglu, Chernozhukov & Yildiz (2016) describe a "folk theorem" of sorts which held that non-common beliefs would eventually converge if they were updated from the same sequence of data. They show the folk theorem need not obtain if players are uncertain about conditional likelihood functions, even if presented with the same sequence of data points. Our paper highlights the possibility that actors may wish to expedite or otherwise manipulate the process of belief convergence by misreporting the data itself.

3 A Model of Appointee-Bureaucrat Interaction

3.1 Preliminaries

We consider a political appointee, A, and a bureaucrat (career civil servant), B. At the start of the game, nature draws a state of the world, $\omega \in \{0, 1\}$, where $\Pr(\omega = 1) = \pi$. The bureaucrat submits a report, $s \in \{0, 1\}$, to the appointee. The appointee then has two tasks: reacting to emergent issues and proactively setting forward-looking agency policy. Reacting to an issue requires a binary response, $r \in \{0, 1\}$, while proactive policy may be more granular, $p \in [0, 1]$.

Both players incur quadratic loss in the lack of congruence between A's reactive policy response and the state of the world as well as in the extent to which A's proactive policy does not match the likelihood of the state of the world being 1. Each weights the congruence of reactive policy by $\rho_i > 0$. The players' preferences may be represented by the vNM utility function:

$$u_i(p, r|\pi, \omega) = -\rho_i \cdot (r - \omega)^2 - (p - \pi)^2, \ i = A, B. \tag{1}$$

The bureaucrat learns the state ω before issuing the report s. Further, B is certain that $\omega \sim Bern(\hat{\pi})$, i.e., that the true value of $\pi := \Pr(\omega = 1)$ is $\hat{\pi}$. The appointee neither knows the distribution of ω nor does she learn the true state of the world at the start of the game. Let A's prior beliefs about the true value of π be distributed according to the beta distribution. Rather than the more common parameterization in terms of α and β, we employ an alternative parameterization in terms of the prior mean, π_0, and concentration, N (sometimes referred to as the sample size in a nod to the binomial distribution for which the beta distribution is a conjugate prior).[5] The mean of the appointee's prior beliefs is a natural quantity of interest, as is N, which represents the precision of those beliefs, as variance may be written as $\frac{\pi_0(1-\pi_0)}{N+1}$. We take $N \geq 2$. Without loss of generality, let $\pi_0 \leq 1/2$. This assumption implies that the appointee believes that the true value of π is likely to favor $\omega = 0$ is (weakly) more likely than $\omega = 1$ before the game begins. This information structure is common knowledge.

As the subscript on ρ suggests, the appointee and bureaucrat may have different weighting parameters. Since the appointee simultaneously and independently chooses r and p, however, the weighting parameter ρ_A does not factor into her decision calculus. As such, we set $\rho_A = 1$ and denote $\rho_B = \rho$.

The game is one of sequential moves and incomplete information, requiring sequential rationality and consistency of beliefs in equilibrium. While sequential rationality takes its usual form, the conditions that must be met for consistent beliefs require more thought in the presence of non-common priors. A set of substantive assumptions about the information structure dictates the way in which A updates her beliefs given her innate disagreement with B.

Specifically, we suppose the appointee agrees to update her beliefs about the underlying probability that the state of the world equals 1 with two stipulations. While the appointee is willing to update from the bureaucrat's reports, A uses her own priors over π. Further, the appointee takes into account the bureaucrat's equilibrium strategy when updating as well as B's beliefs.

[5]Let $Beta(\pi; \alpha, \beta)$ be the traditionally parameterized beta distribution, and define $\pi_0 := \frac{\alpha}{\alpha+\beta}$ and $N := \alpha + \beta$. We may then write $\pi \sim Beta(\pi; \pi_0 N, (1 - \pi_0)N)$.

3.2 Comments on the Model's Features

As noted above, our paper distinguishes itself from traditional cheap talk models by incorporating the two independent policy decisions—reactive and proactive—that a single actor must make upon receiving the bureaucrat's message. This dual nature of decision-making represents a key challenge for political appointees (and executive decision-makers more broadly), balancing immediate response with long-term strategic planning. The impact of a single report on these two policy dimensions can either exacerbate or mitigate the bureaucrat's intent to shape the appointee's beliefs.

With reactive policy responses, we intend to evoke the need for agencies to shift resources around quickly to best deal with evolving situations. The appropriateness of the response to an emergent issue ultimately becomes common knowledge in a way that the appropriateness of structural policy does not. Thus, the appointee's preferences around reactive policy responses depend on the actual state of the world, $\omega \in \{0, 1\}$. In addition to responding to so-called crises of the day, a political appointee is also responsible for setting the direction of the agency. Proactive policy is meant to connote this latter, structural kind of policy. In this endeavor, appointees seek to equip the agency to adeptly respond to the full mix of challenges it is likely to face ($\omega = 1$ and $\omega = 0$) and in the correct proportion (her perception of π).

More substantively, reactive policy is a binary decision and proactive policy is chosen from a continuum because the decisions reflect, respectively, policies that are up or down choices or policies that accommodate (perhaps require) a more nuanced, graduated response. For example, a government's decision to declare a state of emergency due to a natural disaster is binary; it declares an emergency or it does not, based on the immediate situation. Alternatively, budget allocations for disaster preparedness are proactive policies that are not binary. They involve a continuum of funding levels that can be adjusted over time based on perceived risks and available resources, reflecting the continuous nature of proactive policy modeling.

The higher ρ is, the more important is reactive policy relative to proactive policy for the bureaucrat. As an incorrect reactive policy becomes more acceptable to the bureaucrat, $\rho \downarrow 0$.[6] The model does

[6] In an alternative formulation, $\omega_t \sim Bern(\pi)$ independently for all periods t, r is the policy at $t = 0$, and p is the policy for all $t \geq 1$. This embodies the conception of p as structural, forward-looking policy that sets the agency up to deal with the mix of $\omega = 1$ and $\omega = 0$ in the future. The vNM utility function would be

$$-(r - \omega_0) - \sum_{t=1}^{\infty} \delta^t \cdot (p - \omega_t)^2.$$

As will be true in the present formulation, the optimal p is the mean of the belief distribution over the state-generating process. Connecting the two formulations, $\rho = \frac{1-\delta}{\delta}$, such that ρ is still inversely related to the patience of the players in the game. We thank Zhaotian Luo for illuminating this point.

not enforce the same relative weighting on reactive policy, ρ, across different actors. We recognize the distinct time horizons and objectives that appointees and bureaucrats operate under. Yet, as noted above, ρ_A does not enter into the appointee's calculations. Aside from potential (though inconsequential) differences in weight placed on reactive policy relative to proactive policy, the preferences of the appointee and bureaucrat are largely aligned, conditional on the beliefs on which the preferences depend. If they had the same beliefs—not just symmetric knowledge of ω but beliefs about π—their induced preferences would be identical.

We assume that disagreement between bureaucrats and appointees (or advisors and advisees generally) is based on differences in beliefs rather than differences in preferences for several, substantive, reasons. First, in bureaucratic and political environments, individuals often share overarching goals (such as public welfare, national security, environmental protection), which mean that their preferences align—conditional on beliefs. If they had the same beliefs, they would agree on the best policy to pursue their shared goals. Disagreement in their beliefs about the state of the world or the effectiveness of various actions arises due to differences in information, expertise, or interpretative frameworks. By focusing on belief convergence rather than preference alignment, the model attempts to capture the dynamics of policymaking where the key challenge is often not about convincing others to change their objectives but rather about ensuring that decisions are made with the best available information.

Furthermore, to observe a change in preferences within the framework of a model, some learning must occur (short of declaring one of the players changes type mid-game). Beliefs can change within models, while preferences conditional on beliefs customarily do not. The accounts of appointees coming around suggest that learning has occurred, and their behavior is not just compromise but rather a deeper alignment with the bureaucracy they lead. If disagreement stemmed only from having preferences at odds, there would be no abatement in disagreement. If the disagreement is based on belief divergence and if preferences depend non-trivially on beliefs, as is the case in this model, then learning may lead to belief convergence and, indirectly, to more aligned preferences.

We instantiate initial differences in beliefs using non-common priors over π, which best captures our view that belief divergence stems from the different bodies of evidence to which the appointee and the bureaucrat have been privy (Aumann 1976). The empirical, albeit anecdotal, observations that appointee updating does occur, but not all at once, suggests non-common priors may be at play. Further, we are encouraged in this modeling choice by other work within the rational choice tradition engaging with non-common priors (Hirsch 2016, Izzo 2023). The partial reconciliation of priors takes the form of a convergence of beliefs about π. Whether the bureaucrat in fact knows the true value of π to be $\hat{\pi}$ or is instead so certain as to possess essentially degenerate beliefs around $\hat{\pi}$ (i.e., non-degenerate bureaucrat beliefs featuring an arbitrarily large value of N), either interpretation is compatible with the model and notions of bureaucratic expertise.

If an appointee could directly observe the state of the world, we would still witness some degree of coming around, but not by way of any bureaucratic manipulation. Appointee beliefs could very well shift towards the bureaucrat's, and in expectation they would,[7] but the likely belief change would be precisely what we observe in a separating, i.e., truth-telling, equilibrium. It is not coming around, then, that relies on asymmetric information, but rather the possibility that the appointee comes around in response to misreporing, i.e., when she would not have if able to observe the state of the world. The possibility of pooling equilibria, in which no communication occurs, also emerges under asymmetric information; here the appointee does not come around even when they would if able to observe the state of the world.

Why should the appointee should listen to the bureaucrat at all, and then only upon receiving a report. Why is it not possible for the bureaucrat to report on the state of the world from $\{0, 1\}$ and transmit a sincerely held belief about $\hat{\pi}$ from $[0, 1]$? First, the assumption that the bureaucrat sends a single message is less a representation of limited communication from the sender than it is a representation of the receiver's limited ability to commit to using any message to inform just one of the decisions for which the report has relevance.

A second reason to limit the bureaucrat to a single message comes from a consideration of the reactive and proactive policy decisions in isolation. Since the state of the world is only indirectly relevant to the proactive policy decision, it is not a cheap talk setting when taken in isolation. Even if shoe-horned into the form of a traditional cheap-talk model, the equilibrium would be pooling—featuring no communication—since the bureaucrat's payoffs on the proactive policy front do not depend on their private information about the state of the world. It is only through yoking the evolution of the beliefs with proactive policy relevance to the reactive policy setting that it is possible to discipline (borrowing terminology from Farrell & Gibbons (1989)) the proactive policy setting. The possibility of discipline, interestingly, comes with the possibility that the proactive policy setting subverts the separating equilibrium—with full communication—that the reactive policy decision displays when taken in isolation.

[7]Provided the players have no uncertainty about the conditional likelihood functions, beliefs would converge asymptotically (Acemoglu, Chernozhukov & Yildiz 2016). This is the case in our model, since $\Pr(\Pr(\omega = 1) = \pi|\pi) = 1$.

Finally, one might ask why the bureaucrat's message space must also be binary? Might they wish to issue a third sort of report, or omit a report entirely? Since the bureaucrat's private information is binary, any more than two potential messages would be redundant, as the mapping from the set of states to the set of reports would either be one-to-many or fail to be subjective.

In sum, and from a technical point of view, this paper's contribution is to model a cheap talk message that informs a single actor making two decisions. This characterization of the model's novelty suggests a number of other contexts to which we might apply the insights from the equilibrium and comparative statics analyses. When speaking to a moderate co-partisan, how might a progressive member of the Democratic party characterize a policy compromise that bore successful results? Do they admit it moved policy in the direction of all party members' preferences? Or do they offer a negative (and knowingly incorrect) account of the policy's outcome to convince the moderate co-partisan of the underlying need for staunchly progressive policies? Or consider so-called national reckonings. What accounts for the persistence of different beliefs around the reality facing a marginalized group? How did the beliefs begin to converge at long last? Without belaboring the point further, we assert that the underlying mechanisms captured by this model are applicable to a much broader array of changes in politically-relevant beliefs.

4 Analysis of the Model

We employ the solution concept of weak perfect Bayesian equilibrium. We require some notation and preliminary results regarding beliefs before turning to analyze actors' optimal strategies. We then begin the analysis in earnest with the appointee's decisions, given a report from the bureaucrat.

4.1 Beliefs and Strategies

As specified above, the appointee chooses a reactive policy $r \in \{0, 1\}$ and a proactive policy $p \in [0, 1]$. The bureaucrat chooses a report (signal) $s \in \{0, 1\}$.[8] Let $q : \{0, 1\} \longrightarrow [0, 1]$ denote the probability that B accurately reports the state of the world as a function of the state, i.e., $q(\omega) = \Pr(s = \omega)$.

Let A's belief that $\omega = 1$ given a report s be given by $\mu(s) \in [0, 1]$. Denote the p.d.f. of the appointee's updated beliefs about π after receiving s by $\nu(\pi; s)$. Lemma 1 expresses μ and ν in terms of the observed report s and the equilibrium probability $q(\cdot)$ of truthfully reporting the state.

[8] We initially set aside the possibility of A mixing over reactive policies. Ultimately, this possibility will be ruled out by an equilibrium selection assumption we invoke below. Further details appear in text and in the appendix.

Lemma 1. *Given a report s and strategy $q(\cdot)$ from B,*

$$\mu(1) := \frac{q(1)\pi_0}{q(1)\pi_0 + (1-q(0))(1-\pi_0)}; \qquad \mu(0) := \frac{(1-q(1))\pi_0}{(1-q(1))\pi_0 + q(0)(1-\pi_0)}; \tag{2}$$

$$\begin{aligned}\nu(\pi; s) = \mu(s) Beta(\pi; \pi_0 N + 1, (1-\pi_0)N) + \\ (1-\mu(s)) Beta(\pi; \pi_0 N, (1-\pi_0)N + 1).\end{aligned} \tag{3}$$

All proofs may be found in the appendix, and see footnote 5 regarding the parameterization of the beta distribution.

To avoid duplicative and inane equilibria, we consider only those equilibria that satisfy the following assumption:

Assumption 1. $\Pr(\omega = 1|s = 1) := \mu(1) \geq \mu(0) =: \Pr(\omega = 1|s = 0)$.

With Lemma 1 in hand, the assumption above reduces to a requirement that, in equilibrium, $q(1) := \Pr(s = 1|\omega = 1) \geq \Pr(s = 1|\omega = 0) =: 1 - q(0)$, and, equivalently, $q(0) := \Pr(s = 0|\omega = 0) \geq \Pr(s = 0|\omega = 1) =: 1 - q(1)$.

4.2 The Appointee's Problem

The appointee's expected utility function, in terms of beliefs μ and ν, is as follows:

$$\mathbb{E}u_A(p, r|s) = -\rho\left[\mu(s)(1-r)^2 + (1-\mu(s))(0-r)^2\right] - \int_0^1 (\pi - p)^2 \nu(\pi; s) d\pi. \tag{4}$$

Lemma 2 addresses the two elements of A's equilibrium behavior.

Lemma 2. *Given a report from B of s, A's optimal reactive policy response, is $r^* = 1$ if $\mu(s) \geq 1/2$ and $r^* = 0$ if $\mu(s) \leq 1/2$. A's optimal proactive policy is $p^* = \frac{\pi_0 N + \mu(s)}{N+1}$.*

For reactive policy, A will choose $r \in \{0, 1\}$ in accordance with the state that she thinks is most likely given the report she received. For proactive policy, A will minimize the expectation of the squared distance from her policy choice to the true value of π by choosing the mean of the distribution describing her updated beliefs about π. Since ν is a mixture distribution, the mean is simply a weighted average of the means of the component distributions. Both results take as given the quantity $\mu(s)$. With Lemma 1, however, we are able to rewrite the results in terms of B's strategy, $q(\cdot)$.

Per Lemma 2, and with Assumption 1, it may be that A optimally chooses $r = s$ ($\mu(1) > 1/2 > \mu(0)$) or it may be that A optimally chooses some r regardless of the report ($1/2 > \mu(1)$ or $\mu(0) > 1/2$). In the first case, A may be said to be responsive to B's report. We refer to the appointee as unresponsive in the latter case.

4.3 The Bureaucrat's Decision to (Mis-)Report the State

Having established A's optimal proactive and reactive responses to a report, s, we next analyze the bureaucrat's state-contingent decision to tell the truth or lie in that report. In so doing, we condition on beliefs held by the appointee. The beliefs, of course, must ultimately satisfy consistency requirements in equilibrium. Since we may precisely infer the appointee's sequentially rational actions from the beliefs, however, it is convenient to proceed using $\mu(1), \mu(0)$. Supposing first that the appointee is responsive to the bureaucrat's reports, i.e., $\mu(1) > 1/2 > \mu(0)$, B is willing to report truthfully when $\omega = 1$ if:

$$u_B(s=1|\omega=1) \geq u_B(s=0|\omega=1)$$
$$-\rho \cdot (1-1)^2 - \left(\hat{\pi} - \frac{\pi_0 N + \mu(1)}{N+1}\right)^2 \geq -\rho \cdot (0-1)^2 - \left(\hat{\pi} - \frac{\pi_0 N + \mu(0)}{N+1}\right)^2.$$

Rearranging terms, B is willing to report $r = 1$ when $\omega = 1$ when:

$$\rho \geq \left(\frac{N(\hat{\pi} - \pi_0) + (\hat{\pi} - \mu(1))}{N+1}\right)^2 - \left(\frac{N(\hat{\pi} - \pi_0) + (\hat{\pi} - \mu(0))}{N+1}\right)^2.$$

The left-hand side of (5) increases in the importance of reactive policy that is congruent to the state of the world. The terms on the right-hand side together represent, in terms of aligning proactive policy with $\hat{\pi}$, the value to the bureaucrat of reporting $s = 0$ instead of $s = 1$. Each specific term capturing policy loss given a particular report s may be viewed as the square of a convex combination of two policy distances: weight $\frac{N}{N+1}$ is placed on the distance from the bureaucrat's claim for the value of π, $\hat{\pi}$, to the appointee's prior expectation for π, π_0; weight $\frac{1}{N+1}$ is placed on the distance from $\hat{\pi}$ to the posterior belief that the $\omega = 1$ given a report of s, given by $\mu(s)$. Most policy loss from proactive policy incongruence stems from the *ex ante* difference in beliefs, $(\hat{\pi} - \pi_0)$. This fixed component of policy loss increases the more precisely held the prior beliefs of the appointee. To the extent that $\mu(s)$ may approach $\hat{\pi}$, this increases the desirability of reporting s. Indeed, the whole right-hand side is positive if reporting 0 would bring the posterior mean (which by Lemma 2 is the appointee's optimal proactive policy, p^*) closer to $\hat{\pi}$ than reporting 1 would. Truth-telling when $\omega = 1$ would then be optimal only if the benefit of misreporting in terms of belief manipulation did not overwhelm the reactive policy benefit of truth-telling captured by the left-hand side. If reporting 1 would have a more desirable effect on the posterior mean, however, the right-hand side will be negative, the inequality above will hold regardless of the value of the left-hand side, and truth-telling will be optimal for B when the state is $\omega = 1$.

We may further rearrange the right-hand side of the inequality to express it in terms of the difference between the appointee's and bureaucrat's prior means, $\hat{\pi} - \pi_0$. We will employ this difference throughout the rest of the analysis as an exogenous quantity of interest. It encodes both the magnitude and direction of the initial disagreement between A and B, where positive values suggest the bureaucrat wishes to shift the appointee's beliefs of the probability that $\omega = 1$ upwards.

$$\rho \geq \left((\hat{\pi} - \pi_0) + \frac{(\pi_0 - \mu(1))}{N+1}\right)^2 - \left((\hat{\pi} - \pi_0) + \frac{(\pi_0 - \mu(0))}{N+1}\right)^2 \Rightarrow$$
$$\rho \geq 2\,\frac{\mu(1) - \mu(0)}{N+1}\left(\frac{\frac{\mu(1)+\mu(0)}{2} - \pi_0}{N+1} - (\hat{\pi} - \pi_0)\right). \tag{5}$$

The analogous condition for B to report $s = 0$ when $\omega = 0$ is as follows:

$$u_B(s = 0|\omega = 0) \geq u_B(s = 1|\omega = 0) \Rightarrow$$
$$\rho \geq 2\,\frac{\mu(1) - \mu(0)}{N+1}\left((\hat{\pi} - \pi_0) + \frac{\pi_0 - \frac{\mu(1)+\mu(0)}{2}}{N+1}\right). \tag{6}$$

The condition in (6) is nearly identical to that in (5), except for the sign of the expression on the right-hand side. The right-hand sides of 5 and 6 do not depend on the state of the world. Rather, they represent the benefit of reporting $s = 1$ or $s = 0$ with regards to aligning proactive policy with $\hat{\pi}$, respectively.

The right-hand side of expression (6) is increasing in $\hat{\pi} - \pi_0$, since reporting $s = 1$ grows increasingly attractive relative to reporting $s = 0$ in terms of aligning proactive policy as $\hat{\pi} - \pi_0$ grows larger. Opposite of above, if B misreports when the state is 0, B will certainly tell the truth when the state is 1. If B finds it worthwhile to misreport when the state is 1, then B will truthfully report when the state is 0, so—as in the analysis of (5)—B will lie in at most one state of the world.

If B always reports truthfully in both states, then $\Pr(\omega = 1|s = 0) := \mu(0) = 0$ and $\Pr(\omega = 1|s =$

$1) := \mu(1) = 1$. If $\mu(1) < 1$ (resp., $\mu(0) > 0$), B must in equilibrium be mixing over reports when $\omega = 0$ (resp., $\omega = 1$). Mixing requires indifference, and the examination of the conditions for truthful reporting above indicated that B can be indifferent between reporting 1 and 0 in at most one state. In a pooling equilibrium, in which $s = 1$ or $s = 0$ regardless of the state, on-the-path posterior beliefs will be equal to priors.[9] This does preclude one type of pooling equilibrium from occurring given the assumption that A chooses reactive policies responsively, viz., pooling on the report $s = 1$. The appointee's belief that $\omega = 1$ given a report of $s = 1$ would be the prior mean of π_0, which is less than 1/2 by assumption, so it would not be sequentially rational for A to choose $r = 1$. The next result summarizes the insights thus far about the possible equilibrium cases in which A's reactive policy is responsive to B's report.

Lemma 3. *If A's reactive policies are responsive to B's reports, one of the following cases must obtain:*

(i) B truthfully reports the state, such that $s = \omega$,

(ii) B mixes between $s = 1$ and $s = 0$ if $\omega = 1$ and reports $s = 0$ if $\omega = 0$,

(iii) B always reports $s = 0$, or

(iv) B reports $s = 1$ if $\omega = 1$ and mixes between $s = 1$ and $s = 0$ if $\omega = 0$.

The result above presumes that A is responsive to B. If A instead selects a given reactive policy response regardless of B's report, then B's only potential influence over A regards her proactive policy. The bureaucrat's best response given unresponsive reactive policy choices by A is to report $r = 1$ if she wishes to shift A's belief about the distribution of π upwards and $r = 0$ if she prefers a downward shift. Only one of these, however, is compatible with an unresponsive equilibrium.

Lemma 4. *The only unresponsive equilibrium entails B choosing $s = 1$ regardless of the state and A choosing $r = 0$ regardless of the report (case (v), below).*

If B always reports $s = 1$, $1/2 > \mu(1) = \pi_0 > \mu(0) = 0$, such that $r^* = 0$ in all cases. Thus, A is unresponsive, and this constitutes an equilibrium as long as $\hat{\pi} - \pi_0$ is large enough such that A wishes to shift beliefs upwards. If, however, B always reports $s = 0$, $\mu(1) = 1 > 1/2 > \mu(0) = \pi_0$. Under such beliefs, A's optimal reactive policies are responsive to the reports, i.e., $r^* = s$ (even though $s = 1$ is off the equilibrium path).

[9] Off-the-path posterior beliefs for μ will be set at the limit of the mixed-strategy equilibrium that approaches the pooling equilibrium.

4.4 Characterizing the Equilibrium Cases

Putting the results of the preceding subsections together, the following proposition describes the equilibrium cases. The difference in beliefs $\hat{\pi} - \pi_0$ (the signed distance between the true underlying probability according to B and the mean of A's prior beliefs) about the likelihood generating states of the world is one determinant of the landscape of equilibria. The quantity ρ, which captures the weight that B places on reactive relative to proactive policy considerations, also plays a key role in delimiting the various equilibrium cases. The exact thresholds dictating what the terms "sufficiently large" and "sufficiently low" entail in the proposition below may be found in the appendix.

One final note is in order before stating the equilibrium cases. If $\hat{\pi} - \pi_0 > \frac{3/4 - \pi_0}{N+1}$ or $\hat{\pi} - \pi_0 < -\frac{\pi_0/2}{N+1}$, the equilibrium cases are mutually exclusive. If not, however, there is a potential multiplicity of equilibria. We achieve uniqueness across the parameter space with the following refinement:

Assumption 2. *When truth-telling is a feasible equilibrium, it is the equilibrium on which players will coordinate. If not, players will coordinate on a mixed strategy for B, if feasible. Otherwise, B will pool on a single report in equilibrium.*

Proposition 1. *Given $\pi_0 < 1/2$ and Assumptions 1 and 2, the following equilibrium cases obtain.*

(i) For any value of $(\hat{\pi} - \pi_0)$, if ρ is sufficiently large, then B truthfully reports in all states, $\omega = 0, 1$, such that $\mu(1) = 1$ and $\mu(0) = 0$.

If $(\hat{\pi} - \pi_0) < \frac{1/2 - \pi_0}{N+1}$,

(ii) for intermediate values of ρ, B truthfully reports when $\omega = 0$ and mixes between $s = 1$ and $s = 0$ when $\omega = 1$ such that $\mu(1) = 1$ and $\mu(0) = (N+1)\left(\left((\hat{\pi} - \pi_0) + \frac{\pi_0}{N+1}\right) + \sqrt{\left((\hat{\pi} - \pi_0) + \frac{\pi_0 - 1}{N+1}\right)^2 - \rho}\right)$;

(iii) for ρ sufficiently small, B reports $s = 0$ in all states, such that $\mu(1) = 1$ and $\mu(0) = \pi_0$.

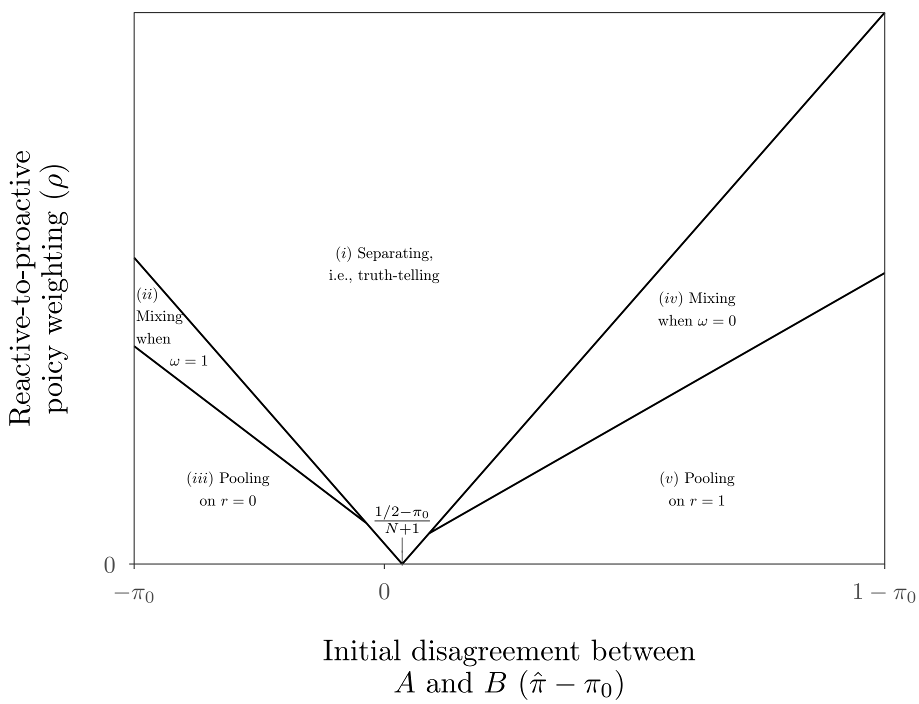
If $(\hat{\pi} - \pi_0) > \frac{1/2 - \pi_0}{N+1}$,

(iv) for intermediate values of ρ, B truthfully reports when $\omega = 1$ and mixes between $s = 1$ and $s = 0$ when $\omega = 0$ such that $\mu(0) = 0$ and $\mu(1) = (N+1)\left(\left((\hat{\pi} - \pi_0) + \frac{\pi_0}{N+1}\right) - \sqrt{\left((\hat{\pi} - \pi_0) + \frac{\pi_0}{N+1}\right)^2 - \rho}\right)$;

(v) for ρ sufficiently small, B reports $s = 1$ in all states, such that $\mu(1) = \pi_0$ and $\mu(0) = 0$.

In all cases, A sets $p^*(s) = \frac{\pi_0 N + \mu(s)}{N+1}$. When B pools on $s = 1$, $r^*(s) = 0$; otherwise, $r^*(s) = s$.

Figure 1: Equilibrium Cases



Notes: The figure depicts the five possible equilibrium cases that obtain across different values of the quantity $\hat{\pi} - \rho_0$, which we refer to as the initial disagreement between the appointee and the bureaucrat, and ρ, which is the weight the bureaucrat places on reactive policy relative to proactive policy. The figure is generated using $\pi_0 = 1/3$ and $N = 6$ (corresponding to $\alpha = 2, \beta = 4$ in the usual parameterization of the beta distribution). The horizontal axis thus tracks increases in $\hat{\pi}$, the bureaucrat's degenerate belief about the data generating process underlying draws of ω, and it ranges from $-1/3$ to $2/3$. The value $(1/2 - \pi_0)/(N + 1)$ corresponds to the value of $\hat{\pi} - \pi_0$ at which the bureaucrat is indifferent between reporting $s = 1$ and $s = 0$ in a separating equilibrium, i.e., willing to separate for any value of ρ.

Figure 1 depicts the equilibrium cases, which include: a "truth-telling" equilibrium (case (i), in which B accurately reports the state, and A chooses $s = r$), "mixing-if-ω" equilibria (cases (ii) and (iv), in which B accurately reports the state in $1 - \omega$ but randomizes between reporting truthfully and not when the state is ω, and A chooses $s = r$), and "pooling-on-s" equilibria (cases (iii) and (v), in which B reports s regardless of the state). Within this last type of equilibrium, A's posterior distribution of beliefs about π are the same as her priors, so her action is determined by the mean of that prior distribution, i.e., π_0, as per Lemma 2. Since π_0 is less than 1/2 by assumption, and as established in Lemmas 3 and 4, A is responsive when B pools on $s = 0$ but unresponsive when B pools on $s = 1$.

The proposition discusses the values of reactive-to-proactive policy weighting (ρ) and the initial disagreement between A and B's beliefs about the likelihood that $\omega = 1$ ($\hat{\pi} - \pi_0$) over which the various equilibrium cases hold. In the first type of equilibrium case, the bureaucrat always accurately reports the state, and the appointee follows this report when choosing policies. This case is possible when the appointee's priors hew towards $\hat{\pi}$ and the bureaucrat's reactive policy considerations are strong enough relative to her long-term considerations. If the bureaucrat's incentives to misrepresent the state are too high, a strategy of truth-telling would not be credible.

As disagreement increases and/or reactive policy considerations carry less weight relative to longer-term alignment, mixing cases become possible, in which the bureaucrat honestly reports one state while occasionally lying in the other state. While the appointee knows the bureaucrat's strategy and belief that $\hat{\pi}$ is the true probability the state is 1, and though she uses her prior distribution to process information, somewhat surprisingly the appointee's beliefs still update in the direction of $\hat{\pi}$ after misreporting in a mixing equilibrium. The mixing equilibria demonstrate the perils of coming around. The appointee's willingness to update her beliefs and the bureaucrat's incentive to misreport lead to the transmission of faulty information and incongruent reactive policy.

Although the action of "coming around" occurs in the mixing cases, the pooling cases are not without interest. If the appointee is so predisposed against the bureaucrat's view that they each deem a different state to be more likely than the other even after B's report (i.e., case (v)), then the appointee will choose policies that directly contradict the bureaucrat's report. This constitutes one explanation for the observation that parties in a disagreement seem to cede less ground the more polarized their positions are relative to one another. If the appointee and the bureaucrat agree on which state is more likely, however, the appointee engages in a different sort of entrenchment (case (iii)). In this case, the appointee will follow the bureaucrat's advice when setting reactive policy to an extent that overstates her confidence that the state is as the bureaucrat says. This equilibrium behavior may provide insight into notions of a vanishing middle, in which even moderate appointees tend to behave similarly to ideological purists.

The next result offers another view of the landscape of equilibrium cases, highlighting comparative statics about the probabilities with which B accurately reports each state of the world and the beliefs A holds consistent with those probabilities. While the statements hold across the parameter space, cases (*ii*) and (*iv*) are the only cases in which there is actual movement in the quantities of interest. As such, these statements may be read primarily as descriptions of the mixing cases.

Corollary 1. *Let all increases/decreases be weak.*
As $(\hat{\pi} - \pi_0)$ *increases or* ρ *increases:*

- *the probability that* B *reports* $s = \omega$ *when* $\omega = 1$ *increases, and*
- *A's belief that* $\omega = 0$ *given* $s = 0$ *increases.*

As $(\hat{\pi} - \pi_0)$ *decreases or* ρ *increases:*

- *the probability that* B *reports* $s = \omega$ *when* $\omega = 0$ *increases, and*
- *A's belief that* $\omega = 1$ *given* $s = 1$ *increases.*

The bureaucrat's truthfulness increases as the value of congruent reactive policy increases, increasing A's belief in B's reports. As $(\hat{\pi} - \pi_0)$ increases from extreme negative values, the bureaucrat has less reason to misreport when $\omega = 1$, since B has less to gain in terms of proactive policy from manipulating A's beliefs. Similarly, as $(\hat{\pi} - \pi_0)$ decreases from extreme positive values, B has less to gain in terms of proactive policy from manipulating A's beliefs. As initial disagreement abates, then, misreporting becomes less frequent and the extent to which A believes B's reports increases. Sufficient *ex ante* agreement emerges as something of a prerequisite for trust in the sense of A's equilibrium willingness to believe B's reports.

Corollary 1 suggests that the elusive concept of "trust" that arises in a truth-telling equilibrium and, to a lesser extent, in a mixing equilibrium is, in fact, a lack of *ex-ante* disagreement. The more similar the views of an appointee to the views within the agency they oversee, the more we expect bureaucrats to report high levels of honesty and the appointee to report high levels of trust. An empirical finding in line with such a prediction would hardly be surprising, however, it would be wrong to interpret higher trust among ideologically-aligned appointees and bureaucrats as necessarily indicating some unobserved relationship between co-partisans. It may be simply the absence of disagreement necessary to motivate misreporting.

While the model is not dynamic, the result above suggests that truth-telling would likely constitute an absorbing state in a repeated setting. Updating the beta distribution in a given period essentially adds 1 to N, where N is inversely related to the malleability of A's beliefs. Further, the benefit of misreporting subsides as A and B's beliefs grow more similar (in expectation). Both of these are favorable conditions for the truth-telling equilibrium case.

The values of $(\hat{\pi} - \pi_0)$ compatible with each equilibrium case depend on both the values of ρ and N, the precision with which the appointee's beliefs are held. We add one additional assumption, namely that ρ is bounded slightly away from 0.[10] This significantly assists in the comparative statics with respect to N.

Assumption 3. *Let ρ be bounded below by $\left(\frac{1-\pi_0}{N+1}\right)^2$.*

Corollary 2. *As ρ or N increases, given $\pi_0 < 1/2$ and assumptions 1-3, the interval of values of $(\hat{\pi} - \pi_0)$ compatible with each of the following equilibrium cases is (according to the order in parentheses):*

(i) increasing (set inclusion),

(ii) decreasing (strong set order),

(iii) decreasing (set inclusion),

(iv) increasing (strong set order),

(v) decreasing (set inclusion).

The more precisely held the incorrect belief of π_0, i.e., the higher is N, the greater room for truth-telling. Firmly held wrong beliefs lead to more truth-telling because there is less to gain in terms of proactive policy from misreporting because less updating is likely to happen. The more certain the appointee's beliefs, the more *ex ante* disagreement is needed to sustain some mixing. Similarly, the greater the weight on appropriate reactive policy, the more extreme the extent of disagreement required to support a mixing equilibrium.

If there is too much agreement, misreporting is not worth the cost of reactive policies that do not match the state. The more certainty appointees possess in their beliefs (higher N), the greater the

[10] This is, for example, a weaker assumption than would be necessary to exclude regions of equilibrium multiplicity.

ex-ante disagreement needs to be to motivate B to change A's beliefs enough to support a pooling equilibrium, even though no coming around ultimately occurs in pooling cases. Conversely, the more malleable the beliefs (lower N) of the appointee, the greater the temptation for the bureaucrat to misreport and the larger the region of the uninformative equilibrium.

5 Discussion

The model sheds light on recurring and historic interactions between political appointees and career civil servants working together to set and inform agency policy. Several such contexts receive attention in this section. The first application concerns the annual budget process. The second subsection turns to the unique period of serious deregulation in U.S. history. Third, the model offers important insights into factual inaccuracies that the government's scientific arm knowingly propagated during the COVID-19 pandemic.

5.1 Budgeting

In the environment of budgeting, reactive policies entail scenarios that require immediate binary decisions to be made in response to specific events—like emergency funding after a disaster. Proactive policies, in contrast, involve strategic anticipation of future scenarios, guiding long-term, granular budgeting decisions for initiatives such as building public health infrastructure. Together, these immediate responses and long-term planning inform budgetary practices.

Continual increases in the size of government, particularly in terms of bureaucratic scope and cost, have been of interest to political economists going back at least as far as Meltzer & Richard (1981). The trend in government expansion is particularly vexing when considering presidential oversight over the bureaucracy. Typically, appointees arrive at the bureaucracy as agents of the president with mandates reflective of the president's priorities, often characterized by a predisposition towards fiscal conservatism in the face of congressional constraints and diverse agency demands. Consequently, it stands to reason that appointees most often begin their tenure with an eye towards reducing spending.

Career civil servants on the other hand, who are deeply enmeshed in the operational realities of their agencies, often recognize a need for expanded agency capabilities. Within our model this perception is quantified as $\hat{\pi}$ approaching 1—indicating a belief in the necessity of greater policy response capacity for the agency. In contrast to the traditional view of bureaucrats as solely budget-maximizing agents

(Niskanen 1971), $\hat{\pi}$ is not borne of a desire for agency largesse but rather reflects a well-founded understanding of the structural policies and resources needed for effective governance in the light of future states of the world. In this context, a bureaucrat may intentionally overstate needs on a particular line-item (reporting $s = 1$ when $\omega = 0$) in spite of, not because of, obtaining excessive funding for that project.

Appointees, initially adhering to the president's fiscal conservatism (π_0 less than 1/2), might view the agency's budget request skeptically. Indeed, if there is too much *ex-ante* disagreement, not only will the bureaucrat pool on a single uninformative signal, but the appointee will choose reactive policy that contradicts the bureaucrat's signal and staunchly refuse to request additional funding for the agency (i.e., a non-responsive equilibrium).

The appointee may reconsider her initial beliefs about π as the bureaucrat provides reports of the emerging challenges and the critical role of the agency in addressing them, especially if the appointee is initially open to the agency's mission. This shift is not a preference change but an evolution of their beliefs based on new information gleaned from the bureaucrat's reports. The result is a mixed equilibrium where appointee-approved budget requests, constituting proactive policy choices, could exceed the initial projections of both the appointee and the president.

Such evolution in understanding, facilitated by partial revelation of information, leads to chief executives routinely facing a Cabinet of high-demanders. Warren (2012), for instance, describes the trajectory of a Reagan-era Commerce Secretary who went from zealous slasher of budgets to ardent defender of "every 35 cents" of the Commerce Department's budget. While the appointee comes around to agency-specific considerations, they may lose sight of the externalities posed by individual budget increases, suggesting that agencies might collectively lean towards over-requesting resources in the aggregate. A tragedy of the commons may result, where each agency seeks more without considering the cumulative impact of overall resources or the president's broader policy objectives.

In this example, $\hat{\pi}$ approaching 1 does not alter the model's logic, but it highlights the role of informed belief change in driving budget decisions. Through this process, the appointee learns about the agency's vital role and how an increased budget supports this important work. The narrative shifts from a discussion about budget increases to an understanding of the agency's crucial function and the resources needed to fulfill it.

While the model delineates reactive and proactive policies as being wholly distinct, we acknowledge the potential permeability of this distinction. Reactive policy responses, such as emergency funding after a natural disaster, may evolve into proactive strategies, like comprehensive disaster preparedness programs, indicating a strategic shift in focus based on the evolving understanding of needs ($\hat{\pi}$ approaching 1). Initial reactions can sow seeds of long-term strategic initiatives, driven by updated beliefs and resource allocations. Future analyses might explore this progression, emphasizing how beliefs and strategies adapt over time.

5.2 Deregulation

Our model is also useful in understanding the limits to the process of coming around, specifically the inability to shift the appointee's beliefs given substantial *ex-ante* disagreement. This dynamic is clearly illustrated in the deregulation efforts of the 1970s and 1980s, which Derthick & Quirk (1985) comprehensively describes. During this era, starting with the Ford and Carter administrations, there was a concerted effort to deregulate several industries, including telecommunications and commercial aviation. To further this policy goal, presidents nominated political appointees to serve on regulatory boards and commissions who supported deregulation. The profound ideological divide between these appointees and bureaucrats, especially as deregulation threatened to reduce or abolish regulatory bodies where these bureaucrats served, is evident in the case of the Federal Communications Commission (FCC) and the now-defunct Civil Aeronautics Board (CAB).

In these agencies, career bureaucrats, drawing from their expertise and foresight of potential downstream impacts, may have sought to temper the appointees' aggressive deregulation agenda. Their approach may have included recommendations about reactive policies, such as highlighting the risks associated with swiftly relaxing existing regulations that governed market entrants (see, for example, (Dempsey 1979, 129-139)), demonstrating a resistance to immediate changes that could destabilize markets. The appointees' strong commitment to deregulation, however, often meant that communication from bureaucrats did little to shift their beliefs.

Appointee beliefs, in the notation of our model, likely featured $\pi_0 < 1/2$ and a large value of N, indicating *ex ante* disagreement with an intransigent appointee. That environment would result in a non-responsive equilibrium, with the civil service pooling on reports of $s = 1$ and the appointee choosing $r = 0$ and $p = \pi_0$; appointees did not update proactive policy in the civil servants' desired direction, nor did they effectively collaborate on reactive policy measures. The gradual replacement of the rank-and-file with bureaucrats sharing similar deregulatory convictions to the appointees offers evidence in favor of this equilibrium being the case at the outset.

What of the received wisdom that deregulation received support from within the agencies themselves? This characterizes the situation after appointees hired like-minded bureaucrats to populate the agencies with civil servants sympathetic to deregulation. Indeed, in many cases, the careerists brought on were even more fervent in their support of deregulation than the political appointees, such that a mixing or truth-telling equilibrium obtained in which the appointees' beliefs were pulled even farther in the direction of deregulation. A notable instance of this influence was when these new bureaucrats used their gained trust to persuade appointees to support an "early sunset" of the CAB, hastening its dissolution earlier than initially planned.

While the complete replacement of the bureaucracy falls outside of our model, it exemplifies the necessity of similar beliefs for the "trust" that underpins the assimilation of appointees. The following passage from Pfiffner (1988, p. 97) also suggests that behavior described as "trust" increases as *ex-ante* beliefs grow more proximate ($|\hat{\pi} - \pi_0|$ decreasing), and it further hints at the usefulness of a more dynamic interpretation of the model:

> Despite their initial distrust of career executives, political appointees usually develop over time a trust for the career executives who work for them. This is a predictable cycle that has operated in all recent presidential administrations, even if not in all political appointees. The cycle is characterized by initial suspicion and hostility, which is followed by two or three years of learning to work together. This results in a more sophisticated appreciation of the contribution of the career service and a mutual respect and trust.

A central goal of this paper has been to understand which political appointees, or rather appointee-agency pairs, find themselves subject to this "predictable cycle." In the case of deregulation, *ex-ante* disagreement was initially too large to support any form of trust, but the replacement of civil servants led to greater *ex-ante* agreement and a working relationship that accelerated deregulation.

Importantly, this discussion raises questions about the nature of coming around across different policy areas. For instance, an appointee's belief about the allocation of agency budgets may be more malleable and subject to assimilation than their beliefs about specific regulatory policies. This could be due to the less ideologically charged nature of budget decisions compared to deeply ingrained beliefs about regulation and government intervention. Moreover, the magnitude of *ex-ante* disagreement between appointees and bureaucrats could vary significantly depending on whether the issue at hand is budgetary or policy-oriented. Budgetary disagreements might represent differences in magnitude—how much funding an agency receives—whereas policy disagreements could be different in kind. Yet our model is flexible enough to accommodate these differences within a parameter space that allow for varying *ex ante* disagreement, appointee certainty, and interpretations of the unidimensional set of policies, highlighting a similar logic underlying ostensibly different contexts.

5.3 Knowing Misrepresentation in COVID-19 Guidance

The scientists at the Centers for Disease Control (CDC) and related centers such as the National Institute of Allergy and Infectious Diseases (NIAID) are, in some sense, the epitome of the devoted civil servant. As in the case of Dr. Anthony Fauci, they are not political appointees, and they often serve in their roles for decades, across administrations. Also like Dr. Fauci, in addition to consistently fighting for greater capacity on behalf of the U.S. medical establishment, these bureaucrat-scientists are also fervent advocates for trust in the scientific community. Especially in the early days of the COVID-19 pandemic, government physicians and scientists practically begged the administration to understand the gravity of the nascent pandemic. Why, then, did the CDC knowingly (albeit rarely) pass along misinformation, or at the very least misleading information?

In early 2021, the CDC stated that outdoor transmission accounted for "less than 10%" of COVID cases. Benchmarking the figure at 10%, however, overstated the facts by one, possibly two orders of magnitude (Leonhardt 2021). Because the CDC placed such high weight on developing a general infrastructure of caution, it found itself misreporting at times when available evidence suggested less caution might be appropriate. It was precisely the desire to move policymakers and citizens to appreciate the ongoing gravity of the pandemic that led them to knowingly, in the notation of the model, report $s = 1$ when $\omega = 0$.

What was the potential reactive-policy loss? Besides the risk of reputational harm, which could be incorporated into the reactive-to-proactive policy weighting, there is the economic and psychological harm from continued austerity measures. The CDC made the decision to risk inappropriate policy regarding outdoor behavior for the sake of continued vigilance in policies and resource allotment going forward in the pandemic.[11]

The complexity of real-world interactions often includes subtler forms of communication like strategic omission and, as in the case of the CDC's guidance on outdoor transmission, ambiguity. Strategic

[11] Another example of valuing proactive policymaking over appropriate reactive policymaking includes early guidance understating the value of mask-wearing. In this case, Dr. Fauci wished to ensure sufficient safety equipment for the nation's medical workers and avoid a run on personal protective equipment. In both examples, the desire to ready the nation for the mix of $\omega = 1$ and $\omega = 0$ events going forward led scientists to misrepresent the facts in a particular instance.

omission involves withholding certain facts or details without providing false information. We address above the implications for the model of incorporating omission as a choice. Because there are two states of the world, any more than two choices of message would entail some redundancy in terms of strategies for the bureaucrat.

Ambiguity, however, entails providing information that is deliberately vague or open to multiple interpretations. This approach can be employed to navigate complicated political issues, where explicit truth or falsehood may be too risky or counterproductive. Ambiguity might allow a bureaucrat to maintain flexibility and adaptability, adjusting their influence on the appointee as situations evolve. While our model does not explicitly address these subtler forms of communication, recognizing their presence and potential impact within bureaucracies is important. Future research could explore how these strategies interact with the mechanisms of truth and falsehood in information transmission and belief manipulation—potentially providing a more comprehensive understanding of bureaucratic communications and implications for policymaking.

6 Conclusion

In the Senate hearings that would ultimately confirm the appointment of former Texas Governor Rick Perry as Secretary of the Department of Energy (DOE), Perry's opening remarks expressed contrition for calling for the abolition of that very department as a presidential candidate some five years earlier. In fact, it seems that Perry once held the DOE in such low esteem at the time that he was unable to recall its name when asked which departments would be on his chopping block if elected president. Yet, in a striking reversal, Perry told the Senate that he had "learned a great deal about the important work being done every day" at the DOE, and "after being briefed on so many of the vital functions of the Department of Energy, I regret recommending its elimination."[12] While the softening of Gov. Perry's stance towards the DOE is notable given his initial disposition toward the department and the minimal information he required to change his mind, his journey is but one in a long tradition of political appointees coming around to the agencies that they are appointed to lead. Usually, though, the process of appointee assimilation is slower, taking place over the years after appointment and confirmation, during which time appointees learn from and eventually advocate for the career civil servants whom they oversee.

This paper provides an account of the process and perils of such assimilation. A bureaucrat

[12] https://www.vox.com/energy-and-environment/2017/1/19/14314296/perry-regret-department-energy

may strategically misrepresent the information she provides to her political appointee when there is significant disagreement between the two or a high value on structural policy. Unless there is extreme disagreement between the appointee and the bureaucrat, the appointee updates her beliefs such that they more closely resemble the bureaucrat's position. This outcome is consistent with the bureaucrat's goal of having the appointee come around more quickly to her position than the appointee might have otherwise done if left alone to update according to confirmations (or refutations) of prior beliefs.

Though somewhat outside the strictest interpretation of the model, one may view the three types of equilibrium cases represent different stages of the process of coming around. Over time the appointee's beliefs will grow more similar to the bureaucrat's. As the disagreement in beliefs (and thus policy preferences) between bureaucrat and appointee decreases, we move from pooling to mixing to separating equilibria, each featuring more truth-telling than the last.

The model also highlights the role of trust within the professional relationship between appointees and bureaucrats. In our model, trust emerges as an equilibrium quantity. Trust in the sense of appointee responsiveness in both reactive and proactive policymaking to reports from the bureaucrat requires a sufficient amount of *ex-ante* alignment.

Further, although bureaucrats may engage in deception, there is little reason to expect them to be completely uninformative at all times. Consistent with the literature and demonstrated by the cases we examined, bureaucrats have ample motivation to issue truthful reports. It would be just as incorrect, however, to assume the bureaucrat will be entirely informative, even in the presence of immediate policy consequences.

The strategic interaction this paper explores may certainly be present in other contexts if the information provided from one actor to another has implications for multiple decisions of varying timelines or scopes. Consider the case of a seller wishing to establish a long-term relationship with a buyer. The seller, knowing the actual time it will take to deliver a product, may claim more rapid delivery. While, in the short-term, the buyer will be disappointed by a later-than-promised delivery, it may be that the promise of quick turnaround may increase the favorability of the buyer towards the seller and bolster the longer-term relationship, with failure to meet the first deadline seen as an aberration rather than an indication of the seller's reliability. Of course, as detailed above, this requires sufficient trust to be present at the outset of the relationship. Otherwise, no such belief manipulation would be possible.

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Proofs

Lemma 1. *Given a report s and strategy $q(\cdot)$ from B,*

$$\mu(1) := \frac{q(1)\pi_0}{q(1)\pi_0 + (1-q(0))(1-\pi_0)}; \qquad \mu(0) := \frac{(1-q(1))\pi_0}{(1-q(1))\pi_0 + q(0)(1-\pi_0)};$$

$$\begin{aligned}\nu(\pi; s) = \mu(s)Beta(\pi; \pi_0 N + 1, (1-\pi_0)N) +\\ (1-\mu(s))Beta(\pi; \pi_0 N, (1-\pi_0)N + 1).\end{aligned}$$

Proof of Lemma 1. For ease of exposition in the following proof, we employ the (α, β) parameterization of the Beta distribution, where $\pi_0 := \alpha/(\alpha+\beta)$ and $N := \alpha + \beta$, translating back into the (π_0, N) parameterization after deriving the results. Recalling that $q(\omega) = \Pr(s = \omega)$ and $\mu(s) := \Pr(\omega = 1|s)$, we must account for A's uncertainty around π when calculating $\mu(1)$ and $\mu(0)$.

$$\begin{aligned}\Pr(\omega = 1|s = 1) &= \frac{\int_0^1 [q(1)\pi]\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\pi^{\alpha-1}(1-\pi)^{\beta-1}d\pi}{\int_0^1 [q(1)\pi + (1-q(0))(1-\pi)]\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\pi^{\alpha-1}(1-\pi)^{\beta-1}d\pi}\\ &= \frac{q(1)\Gamma(\alpha+1)\Gamma(\beta)}{q(1)\Gamma(\alpha+1)\Gamma(\beta) + (1-q(0))\Gamma(\alpha)\Gamma(\beta+1)}\\ &= \frac{q(1)\alpha}{q(1)\alpha + (1-q(0))\beta} = \frac{q(1)\pi_0}{q(1)\pi_0 + (1-q(0))(1-\pi_0)}.\end{aligned}$$

$$\begin{aligned}\Pr(\omega = 1|s = 0) &= \frac{\int_0^1 [(1-q(1))\pi]\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\pi^{\alpha-1}(1-\pi)^{\beta-1}d\pi}{\int_0^1 [(1-q(1))\pi + q(0)(1-\pi)]\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\pi^{\alpha-1}(1-\pi)^{\beta-1}d\pi}\\ &= \frac{(1-q(1))\Gamma(\alpha+1)\Gamma(\beta)}{(1-q(1))\Gamma(\alpha+1)\Gamma(\beta) + q(0)\Gamma(\alpha)\Gamma(\beta+1)}\\ &= \frac{(1-q(1))\alpha}{(1-q(1))\alpha + q(0)\beta} = \frac{(1-q(1))\pi_0}{(1-q(1))\pi_0 + q(0)(1-\pi_0)}\end{aligned}$$

To calculate the pdf describing A's updated beliefs, $\nu(\pi; 1)$, we conduct a similar calculation as above:

$$\begin{aligned}\nu(\pi|s = 1) &= \frac{[\pi q(1) + (1-\pi)(1-q(0))]\pi^{\alpha-1}(1-\pi)^{\beta-1}}{\int_0^1 [\tilde{\pi} q(1) + (1-\tilde{\pi})(1-q(0))]\tilde{\pi}^{\alpha-1}(1-\tilde{\pi})^{\beta-1}d\pi}\\ &= \frac{q(1)\pi^{\alpha}(1-\pi)^{\beta-1} + (1-q(0))\pi^{\alpha-1}(1-\pi)^{\beta}}{q(1)\frac{\Gamma(\alpha+1)\Gamma(\beta)}{\Gamma(\alpha+1+\beta)} + (1-q(0))\frac{\Gamma(\alpha)\Gamma(\beta+1)}{\Gamma(\alpha+1+\beta)}}.\end{aligned}$$

We may make two useful simplifications of the previous expression:

$$
\begin{aligned}
\pi &\sim \frac{q(1)\alpha}{q(1)\alpha+(1-q(0))\beta}Beta(\alpha+1,\beta)+\frac{(1-q(0))\beta}{q(1)\alpha+(1-q(0))\beta}Beta(\alpha,\beta+1)\\
&=\mu(1)Beta(\alpha+1,\beta)+(1-\mu(1))Beta(\alpha,\beta+1)\\
&=\mu(1)Beta(\pi_0N+1,(1-\pi_0)N)+(1-\mu(1))Beta(\pi_0N,(1-\pi_0)N+1).
\end{aligned}
$$

Performing the same calculations for $\nu(\pi;0)$:

$$
\begin{aligned}
\nu(\pi|s=0) &= \frac{[\pi(1-q(1))+(1-\pi)q(0)]\pi^{\alpha-1}(1-\pi)^{\beta-1}}{\int_0^1[\tilde{\pi}(1-q(1))+(1-\tilde{\pi})q(0)]\tilde{\pi}^{\alpha-1}(1-\tilde{\pi})^{\beta-1}d\pi}\\
&=\frac{(1-q(1))\pi^{\alpha}(1-\pi)^{\beta-1}+q(0)\pi^{\alpha-1}(1-\pi)^{\beta}}{(1-q(1))\frac{\Gamma(\alpha+1)\Gamma(\beta)}{\Gamma(\alpha+1+\beta)}+q(0)\frac{\Gamma(\alpha)\Gamma(\beta+1)}{\Gamma(\alpha+1+\beta)}}.
\end{aligned}
$$

We again make two useful simplifications of the previous expression:

$$
\begin{aligned}
\pi &\sim \frac{(1-q(1))\alpha}{(1-q(1))\alpha+q(0)\beta}Beta(\alpha+1,\beta)+\frac{q(0)\beta}{(1-q(1))\alpha+q(0)\beta}Beta(\alpha,\beta+1)\\
&=\mu(0)Beta(\alpha+1,\beta)+(1-\mu(0))Beta(\alpha,\beta+1)\\
&=\mu(0)Beta(\pi_0N+1,(1-\pi_0)N)+(1-\mu(0))Beta(\pi_0N,(1-\pi_0)N+1).
\end{aligned}
$$

■

Lemma 2. *Given a report from B of s, A's optimal reactive policy response, is $r^*=1$ if $\mu(s)\geq 1/2$ and $r^*=0$ if $\mu(s)\leq 1/2$. A's optimal proactive policy is $p^*=\frac{\pi_0N+\mu(s)}{N+1}$.*

Proof of Lemma 2. For reactive policy responses, A solves for $r^*=\arg\max_{r\in\{0,1\}}-\mu(s)(r-1)^2-(1-\mu(s))(r-0)^2$. Setting $r=1$ achieves a higher (resp., lower) value than $s=0$ if $\mu(s)\geq 1/2$ (resp., $\leq$).

For proactive policy, A solves for $p^*=\arg\max_{p\in[0,1]}\int_0^1-(\pi-p)^2\nu(\pi)d\pi$. Setting p equal to the mean of A's posterior distribution over π minimizes the squared difference. The mean of a mixture distribution is simply the weighted sum of the means of the component distributions, where the weights are inherited from the mixture. Given a report of s, the mean is thus $\mu(s)\frac{\pi_0N+1}{N+1}+(1-\mu(s))\frac{\pi_0N}{N+1}$. Simplifying, we have $p^*=\frac{\pi_0N+\mu(s)}{N+1}$. ■

Lemma 3. *If A's reactive policies are responsive to B's reports, one of the following cases must obtain:*

(i) *B truthfully reports the state, such that $s = \omega$,*

(ii) *B mixes between $s = 1$ and $s = 0$ if $\omega = 1$ and reports $s = 0$ if $\omega = 0$,*

(iii) *B always reports $s = 0$, or*

(iv) *B reports $s = 1$ if $\omega = 1$ and mixes between $s = 1$ and $s = 0$ if $\omega = 0$.*

Proof of Lemma 3. We first formalize the arguments in the text that B will mix in, at most, one of the states. It will be useful to assign some notation to these expressions that encode the value in terms of proactive policy of reporting $r = 1$ instead of $r = 0$ (Δ_1) or the opposite (Δ_0) in terms of appointee beliefs:

$$\Delta_1(\mu(1), \mu(0)) := \left(\hat{\pi} - \frac{\alpha + \mu(0)}{\alpha + \beta + 1}\right)^2 - \left(\hat{\pi} - \frac{\alpha + \mu(1)}{\alpha + \beta + 1}\right)^2 = -\Delta_0(\mu(1), \mu(0)).$$

Recall that the assumption of a responsive equilibrium in the premise of the Lemma implies that $r^* = s$ and requires that

$$\mu(1) > 1/2 > \mu(0). \tag{7}$$

Lemma 5. *In any equilibrium, B will misreport in at most one state.*

Proof of Lemma 5. Suppose otherwise by way of contradiction. The premise implies that B is willing – given beliefs $\mu(1), \mu(0)$ – to misreport when $\omega = 0$, so it must be that $\rho \leq \Delta_1(\mu(1), \mu(0)) \Rightarrow \Delta_1(\mu(1), \mu(0)) > 0$. The premise also implies that B is willing to misreport when $\omega = 1$, so it must also be true that $\rho \leq \Delta_0(\mu(1), \mu(0)) \Rightarrow \Delta_0(\mu(1), \mu(0)) > 0$. Since $\Delta_1(\mu(1), \mu(0)) = -\Delta_0(\mu(1), \mu(0))$ by definition, however, both cannot be strictly greater than 0, providing the contradiction we sought. ■

We may then have five possible equilibrium cases – fully separating, mixing when $\omega = 1$, mixing when $\omega = 0$, pooling on $s = 0$, and pooling on $s = 1$ – each of which we investigate, in turn.

The conditions for a fully-separating equilibrium case are easiest to specify. Since B will be truthfully reporting ($q(1) = q(0) = 1$), A's consistent beliefs will be $\mu(1) = 1$ and $\mu(0) = 0$ (which satisfies (7)).

Mixing when $\omega = 1$ implies that $q(1) \in (0, 1)$ and – with Lemma 5 – that $q(0) = 1$. A's consistent beliefs will be $\mu(1) = 1$ and $\mu(0) \in (0, \pi_0)$ (satisfying (7)), where $\mu(0) = \frac{(1-q(1))\pi_0}{(1-q(1))\pi_0 + (1-\pi_0)}$.

For pooling on $s = 0$ in which A is responsive, choosing $r = 0$ after all on-the-equilibrium-path reports of $s = 0$ and choosing $r = 1$ after an off-the-path report of $s = 1$, A's consistent beliefs must be $\mu(0) = \frac{\alpha}{\alpha+\beta}$ and (to satisfy (7)) $\mu(1) \geq 1/2$. We will uncontroversially set $\mu(1) = 1$ as it is the natural continuation of nearby mixed-strategy equilibria.

Mixing when $\omega = 0$ implies that $q(0) \in (0, 1)$ and – with Lemma 5 – that $q(1) = 1$. A's consistent beliefs will be $\mu(0) = 0$ and $\mu(1) = \frac{\pi_0}{\pi_0+(1-q(0))(1-\pi_0)} < 1$. To satisfy (7), however, it must be that $\mu(1) \geq 1/2$, implying a stronger condition on $\Pr(s = \omega|\omega = 0)$, viz., $q(0) \geq \frac{1-2\pi_0}{1-\pi_0}$.

The bounds on the case above make clear that there cannot be a pooling on $s = 1$ case in which A is responsive, i.e., that satisfies (7). Such a case would require $\mu(0) = 0$ and $\mu(1) = \pi_0$, as posteriors would simply be equal to priors, but by the assumption of $\pi_0 < 1/2$, this implies $\mu(1) < 1/2$, violating (7). ■

Lemma 4. *The only unresponsive equilibrium entails B choosing $s = 1$ regardless of the state and A choosing $r = 0$ regardless of the report.*

Proof of Lemma 4. We have defined A to be unresponsive to B's report if $r^* \neq s$. Given Assumption 1 and Lemma 5, two sets of beliefs may potentially underlie such an equilibrium, viz., $1 = \mu(1) > \mu(0) > 1/2$ and $1/2 > \mu(1) > \mu(0) = 0$. In the former case, A optimally chooses $r = 1$ after either signal, while in the latter case A optimally chooses $r = 0$ after either signal.

In neither case does the effect on reactive policy enter B's calculation about which signal to issue. As such, the bureaucrat's decision is based entirely on the sign of $\Delta_1(\mu(1), \mu(0))$. If it is positive, B would pool on $s = 1$, implying $\mu(1) = 1$ and $\mu(0) = \pi_0$. Yet we assume $\pi_0 < 1/2$, so there cannot exist an unresponsive equilibrium in which B pools on $s = 1$. If $\Delta_1(1, \pi_0)$ is positive, the *responsive* pooling equilibrium detailed in the statement and proof of Lemma 3 obtains.

If $\Delta_0(\mu(1), \mu(0))$ is positive, B would pool on $s = 0$, implying $\mu(0) = 0$ and $\mu(1) = \pi_0 < 1/2$, satisfying the necessary conditions for A to be unresponsive. If it is also the case that $\Delta_0(\pi_0, 0)$ is positive, then an unresponsive equilibrium in which $s = 0$ may obtain. This is the only viable case of an unresponsive equilibrium. ■

Proposition 1. *Given $\pi_0 < 1/2$ and Assumptions 1 and 2, the following equilibrium cases obtain.*

(i) For any value of $(\hat{\pi} - \pi_0)$, if ρ is sufficiently large, then B truthfully reports in all states, $\omega = 0, 1$, such that $\mu(1) = 1$ and $\mu(0) = 0$.

If $(\hat{\pi} - \pi_0) < \frac{1/2-\pi_0}{N+1}$, *(ii) for intermediate values of ρ, B truthfully reports when $\omega = 0$ and mixes between $s = 1$ and $s = 0$ when $\omega = 1$ such that $\mu(1) = 1$ and $\mu(0) = (N+1)\left(\left((\hat{\pi} - \pi_0) + \frac{\pi_0}{N+1}\right) + \sqrt{\left((\hat{\pi} - \pi_0) + \frac{\pi_0 - 1}{N+1}\right)^2 - \rho}\right)$;*

(iii) for ρ sufficiently small, B reports $s = 0$ in all states, such that $\mu(1) = 1$ and $\mu(0) = \pi_0$.

If $(\hat{\pi} - \pi_0) > \frac{1/2 - \pi_0}{N+1}$,

(iv) for intermediate values of ρ, B truthfully reports when $\omega = 1$ and mixes between $s = 1$ and $s = 0$ when $\omega = 0$ such that $\mu(0) = 0$ and $\mu(1) = (N+1)\left(\left((\hat{\pi} - \pi_0) + \frac{\pi_0}{N+1}\right) - \sqrt{\left((\hat{\pi} - \pi_0) + \frac{\pi_0}{N+1}\right)^2 - \rho}\right)$;

(v) for ρ sufficiently small, B reports $s = 1$ in all states, such that $\mu(1) = \pi_0$ and $\mu(0) = 0$.

In all cases, A sets $p^*(s) = \frac{\pi_0 N + \mu(s)}{N+1}$. When B pools on $s = 1$, $r^*(s) = 0$; otherwise, $r^*(s) = s$.

Proof of Proposition 1. Recall the RHS of (5) is the benefit of reporting $s = 0$ for proactive policymaking, denoted:

$$\Delta_0(\mu(1), \mu(0)) := 2\frac{\mu(1) - \mu(0)}{N+1}\left(\frac{\frac{\mu(1)+\mu(0)}{2} - \pi_0}{N+1} - (\hat{\pi} - \pi_0)\right).$$

The RHS of (6) is the benefit of reporting $s = 1$ for proactive policymaking, denoted:

$$\Delta_1(\mu(1), \mu(0)) := 2\frac{\mu(1) - \mu(0)}{N+1}\left((\hat{\pi} - \pi_0) + \frac{\pi_0 - \frac{\mu(1)+\mu(0)}{2}}{N+1}\right).$$

Lemma 2 gives A's optimal proactive and reactive policies. Given those results and in conjunction with the assumptions, Lemmas 3 and 4 outline five viable equilibrium cases. Below we derive the beliefs implied by the strategies entailed in each case (see also the proof of Lemmas 3 and 4) as well as the region of the parameter space over which each case obtains:

(i) Recalling $\mu(s)$ denotes A's belief about the probability that $\omega = 1$ given a report of s, a fully separating equilibrium implies $\mu(1) = 1$ and $\mu(0) = 0$. It must also be the case that

$$\rho \geq \Delta_1(1, 0) \;\; \& \;\; \rho \geq \Delta_0(1, 0),$$

which we note is always possible when $\Delta_1(1, 0) = \Delta_0(1, 0) = 0$, i.e., $(\hat{\pi} - \pi_0) = \frac{1/2 - \pi_0}{N+1}$. The conditions on ρ may be written as follows:

$$\rho \geq \left|\frac{2}{N+1}\left((\hat{\pi} - \pi_0) + \frac{\pi_0 - \frac{1}{2}}{N+1}\right)\right|.$$

Given any $(\hat{\pi} - \pi_0)$, let $\tilde{\rho} := \left| \frac{2}{N+1} \left((\hat{\pi} - \pi_0) + \frac{\pi_0 - \frac{1}{2}}{N+1} \right) \right|$, such that $\forall \rho \geq \tilde{\rho}$, i.e., ρ sufficiently large, a truth-telling equilibrium exists.

(ii) For a mixing-when-$\omega = 1$ case, we must have $\mu(1) = 1$, $\mu(0) = \frac{(1-q(1))\pi_0}{(1-q(1))\pi_0 + (1-\pi_0)}$, and $\rho = \Delta_0(1, \mu(0))$, a condition that implies the benefit to B of reporting $s = 0$ in terms of A's proactive policy is high enough to induce misreporting when $\omega = 1$. We thus have two equations for our two unknowns, $q(1), \mu(0)$, though expressing B's mixing probability when $\omega = 1$, $q(1)$, solely in terms of exogenous elements of the model is less useful than expressing the quantity in terms of $\mu(0)$: $q(1) = \frac{\pi_0 - \mu(0)}{(1-\mu(0))\pi_0}$, a form that prompts us to recall that $\mu(0) \in (0, \pi_0)$. The equations for the beliefs – along with the implication of Assumption 2 that $\rho < \Delta_0(1, 0)$, i.e., that a mixing equilibrium obtains only when no truth-telling equilibrium is possible – do allow us to derive an expression for $\mu(0)$ in terms of exogenous elements of the model.[13]

$$\rho = 2\frac{1 - \mu(0)}{N+1} \left(\frac{\frac{1+\mu(0)}{2} - \pi_0}{N+1} - (\hat{\pi} - \pi_0) \right) \Rightarrow$$

$$\mu(0) = (N+1) \left(\left((\hat{\pi} - \pi_0) + \frac{\pi_0}{N+1} \right) + \sqrt{\left((\hat{\pi} - \pi_0) + \frac{\pi_0 - 1}{N+1} \right)^2 - \rho} \right)$$

The conditions that $0 < \mu(0) < \pi_0$ imply that $\Delta_0(1, \pi_0) \leq \rho < \Delta_0(1, 0)$, the latter of which guarantees the determinant in the expression for $\mu(0)$ is positive. It is easy to observe that $\Delta_0(1, 0)$ is steeper than $\Delta_0(1, \pi_0)$. Asking when the former is greater than the latter, we arrive at the following interval of $(\hat{\pi} - \pi_0)$ over which this region is nonempty is $[-\pi_0, -\frac{\pi_0/2}{N+1})$, the upper bound of which is always strictly greater than the lower bound and always strictly less than $\frac{1/2 - \pi_0}{N+1}$. Taken together, we conclude that for a given $(\hat{\pi} - \pi_0) < \frac{1/2 - \pi_0}{N+1}$, a mixing-when-$\omega = 1$ equilibrium obtains for values of ρ that are intermediate in the sense of lying in the interval $[\Delta_0(1, \pi_0), \Delta_0(1, 0))$.

(iii) For a pooling-on-$s = 0$ equilibrium to obtain, we must have $\mu(0) = \pi_0$. Further, we have the off-path assumption of $\mu(1) = 1$ as discussed in-text as well as the assumption that no truth-telling or mixing equilibrium obtains in the same parameter space, implying $\rho < \min\{\Delta_0(1, \pi_0), \Delta_0(1, 0)\} \Rightarrow$

$$\rho < \min \left\{ \frac{1 - \pi_0}{N+1} \left(\frac{1 - \pi_0}{N+1} - 2(\hat{\pi} - \pi_0) \right), \frac{1}{N+1} \left(\frac{1 - 2\pi_0}{N+1} - 2(\hat{\pi} - \pi_0) \right) \right\},$$

[13] Assumption 2 rules out the other solution to the quadratic.

such that for ρ sufficiently small the pooling-on-$s = 0$ case obtains as long as $(\hat{\pi} - \pi_0) < \frac{1/2-\pi_0}{N+1}$.

(iv) For a mixing-when-$\omega = 0$ case, we must have $\mu(0) = 0$, $\mu(1) = \frac{\pi_0}{\pi_0+(1-q(0))(1-\pi_0)}$, and $\rho = \Delta_1(\mu(1), 0)$, a condition that implies the benefit to B of reporting $s = 0$ in terms of A's proactive policy is high enough to induce misreporting when $\omega = 0$. We thus have two equations for our two unknowns, $q(0), \mu(1)$, though expressing B's mixing probability when $\omega = 0$, $q(0)$, solely in terms of exogenous elements of the model is less useful than expressing the quantity in terms of $\mu(1)$: $q(0) = \frac{\mu(1)-\pi_0}{\mu(1)(1-\pi_0)}$, a form that prompts us to recall that $\mu(1) \in (\pi_0, 1)$. In fact, the lower bound on $\mu(1)$ in a mixing-when-$\omega = 0$ equilibrium must be 1/2, as discussed extensively in and around Lemmas 3 and Lemmas 4. The equations for the beliefs – along with the implication of Assumption 2 that $\rho < \Delta_1(1, 0)$, i.e., that a mixing equilibrium obtains only when no truth-telling equilibrium is possible – do allow us to derive an expression for $\mu(1)$ in terms of exogenous elements of the model.[14]

$$\rho = 2\frac{\mu(1)}{N+1}\left((\hat{\pi} - \pi_0) + \frac{\pi_0 - \frac{\mu(1)}{2}}{N+1}\right) \Rightarrow$$

$$\mu(1) = (N+1)\left(\left((\hat{\pi} - \pi_0) + \frac{\pi_0}{N+1}\right) - \sqrt{\left((\hat{\pi} - \pi_0) + \frac{\pi_0}{N+1}\right)^2 - \rho}\right)$$

The conditions that $1/2 < \mu(0) < 1$ imply that $\Delta_1(1/2, 0) \leq \rho < \Delta_1(1, 0)$, the latter of which guarantees the determinant in the expression for $\mu(0)$ is positive. It is easy to observe that $\Delta_1(1, 0)$ is steeper than $\Delta_0(1/2, 0)$. Asking when the former is greater than the latter, we arrive at the following interval of $(\hat{\pi} - \pi_0)$ over which this region is nonempty is $[\frac{3/4-\pi_0}{N+1}, 1-\pi_0]$, the lower bound of which is always strictly greater than $\frac{1/2-\pi_0}{N+1}$ and strictly less than $1-\pi_0$. Taken together, we conclude that for a given $(\hat{\pi} - \pi_0) > \frac{1/2-\pi_0}{N+1}$, a mixing-when-$\omega = 0$ equilibrium obtains for values of ρ that are intermediate in the sense of lying in the interval $[\Delta_0(1/2, 0), \Delta_0(1, 0))$.

(v) For a pooling-on-$s = 1$ equilibrium to obtain, we must have $\mu(1) = \pi_0$. Further, we have the off-path assumption of $\mu(0) = 0$ as discussed in-text as well as the assumption that no truth-telling or mixing equilibrium obtains in the same parameter space, implying $\rho < \min\{\Delta_1(1/2, 0), \Delta_1(1, 0)\} \Rightarrow$

$$\rho < \min\left\{\frac{1}{N+1}\left((\hat{\pi} - \pi_0) + \frac{\pi_0 - \frac{1}{4}}{N+1}\right), \frac{2}{N+1}\left((\hat{\pi} - \pi_0) + \frac{\pi_0 - \frac{1}{2}}{N+1}\right)\right\},$$

[14] Assumption 2 rules out the other solution to the quadratic.

such that for ρ sufficiently small the pooling-on-$s = 1$ case obtains as long as $(\hat{\pi} - \pi_0) > \frac{1/2 - \pi_0}{N+1}$. This condition also guarantees that B wishes to report $s = 1$ given unresponsive behavior from A, viz., that $\Delta_0(\pi_0, 0)$ is positive.

The final element of the proof is to demonstrate that an equilibrium in which $\mu(1) = 1/2, \mu(0) = 0$ and A randomizes between being responsive and unresponsive would not receive precedent from Assumption 2 over case (v). We would require $\Delta_1(1/2, 0) = \ell \cdot \rho$, where ℓ is the probability that A listens and reacts accordingly to a B mixing-when-$\omega = 0$. To obtain over the region where case (v) obtains, we require when $\Delta_1(1/2, 0) > \rho$ that $\ell \cdot \rho = \Delta_1(1/2, 0)$, but since $\ell < 1$, this implies $\Delta_1(1/2, 0) > \rho = \Delta_1(1/2, 0)/\ell > \Delta_1(1/2, 0)$, a contradiction. ■

Corollary 1. *Let all increases/decreases be weak.*
As $(\hat{\pi} - \pi_0)$ *increases or* ρ *increases:*

- *the probability that* B *reports* $s = \omega$ *when* $\omega = 1$ *increases, and*
- *A's belief that* $\omega = 0$ *when* $s = 0$ *increases.*

As $(\hat{\pi} - \pi_0)$ *decreases or* ρ *increases:*

- *the probability that* B *reports* $s = \omega$ *when* $\omega = 0$ *increases, and*
- *A's belief that* $\omega = 1$ *when* $s = 1$ *increases.*

Proof of Corollary 1. It is, in fact, easier to begin with the effects on the equilibrium beliefs that the state is 1 and then progress to analyze the effects on the mixing probability that would have led to that. First, however, we derive way in which $q(1), q(0)$ depend, respectively, in equilibrium on $\mu(0), \mu(1)$. Intuitively, however, the more B lies about the state being 1, the less believable a report of 0, and analogously for lying about 0 and the reliability of $s = 1$, and these monotonic relationships can be inverted:

$$
\begin{aligned}
q(1) &= \frac{\pi_0 - \mu(0)}{(1 - \mu(0))\pi_0} \\
\frac{\partial q(1)}{\partial \mu(0)} &= \frac{-(1 - \mu(0))\pi_0 - (\pi_0 - \mu(0))(-\pi_0)}{((1 - \mu(0))\pi_0)^2} \\
&= \frac{-\pi_0(1 - \pi_0)}{((1 - \mu(0))\pi_0)^2} < 0
\end{aligned}
$$

$$
\begin{aligned}
q(0) &= \frac{\mu(1) - \pi_0}{\mu(1)(1-\pi_0)} \\
\frac{\partial q(0)}{\partial \mu(1)} &= \frac{\mu(1)(1-\pi_0) - (\mu(1)-\pi_0)(1-\pi_0)}{(\mu(1)(1-\pi_0))^2} \\
&= \frac{\pi_0(1-\pi_0)}{(\mu(1)(1-\pi_0))^2} > 0
\end{aligned}
$$

Moving northeast in the parameter space as depicted in Figure 1, we observe $\mu(0)$ decrease from π_0 to 0 (meaning the belief that $\omega = 0$ when $s = 0$ increases, as stated in the proposition), implying $q(1)$ is increasing (the truthful reporting of $\omega = 1$). The only non-trivial case to check is case (ii) and then only with respect to $(\hat{\pi} - \pi_0)$ as $\mu(0)$ is clearly decreasing in ρ.

$$
\begin{aligned}
&\frac{\partial \mu(0)}{\partial(\hat{\pi} - \pi_0)} = (N+1)\left(1 + \frac{2((\hat{\pi}-\pi_0) + \frac{\pi_0 - 1}{N+1})}{\sqrt{2\left((\hat{\pi}-\pi_0) + \frac{\pi_0-1}{N+1}\right)^2 - \rho}}\right) < 0 \quad \text{since in case } (ii) \\
&(\hat{\pi} - \pi_0) < \frac{1/2 - \pi_0}{N+1} \Rightarrow (\hat{\pi}-\pi_0) < \frac{1-\pi_0}{N+1} \Rightarrow 0 < \frac{1-\pi_0}{N+1} - (\hat{\pi}-\pi_0) \Rightarrow \\
&\left[\frac{1-\pi_0}{N+1} - (\hat{\pi}-\pi_0)\right]^2 > \left[\frac{1-\pi_0}{N+1} - (\hat{\pi}-\pi_0)\right]^2 - \rho \quad (\text{note RHS is } > 0) \Rightarrow \\
&1 < \frac{\frac{1-\pi_0}{N+1} - (\hat{\pi}-\pi_0)}{\sqrt{\left[\frac{1-\pi_0}{N+1} - (\hat{\pi}-\pi_0)\right]^2 - \rho}} \Rightarrow \\
&0 > (N+1)\left[1 + \frac{(\hat{\pi}-\pi_0) + \frac{\pi_0-1}{N+1}}{\sqrt{\left((\hat{\pi}-\pi_0) + \frac{\pi_0-1}{N+1}\right)^2 - \rho}}\right].
\end{aligned}
$$

Moving northwest in the same parameter space, we observe $\mu(1)$ increasing from π_0 to 1 (i.e., decreasing in $(\hat{\pi} - \pi_0)$ and increasing in ρ), implying greater truth-telling when $\omega = 0$ and thus more believable reports of $s = 1$. Again, the only non-trivial case is case (iv), though, and then only with respect to $(\hat{\pi} - \pi_0)$ as $\mu(1)$ is clearly increasing in ρ.

$$
\frac{\partial \mu(1)}{\partial(\hat{\pi} - \pi_0)} = (N+1)\left(1 - \frac{(\hat{\pi}-\pi_0) + \frac{\pi_0}{N+1}}{\sqrt{\left((\hat{\pi}-\pi_0) + \frac{\pi_0}{N+1}\right)^2 - \rho}}\right)
$$

Because the numerator of the fraction on the RHS is positive (in case (iv) $(\hat{\pi} - \pi_0) > \frac{1/2-\pi_0}{N+1} \Rightarrow (\hat{\pi} - \pi_0) + \frac{\pi_0}{N+1} > 0$), the fraction itself is greater than 1, implying the expression as a whole is less than 0. ■

Corollary 2. *As ρ or N increases, given $\pi_0 < 1/2$ and assumptions 1-3, the interval of values of $(\hat{\pi} - \pi_0)$ compatible with each of the following equilibrium cases is (according to the order in parentheses):*

(i) *increasing (set inclusion),*

(ii) *decreasing (strong set order),*

(iii) *decreasing (set inclusion),*

(iv) *increasing (strong set order),*

(v) *decreasing (set inclusion).*

Proof of Corollary 2. Assumptions 1 and 2 narrow the equilibrium cases down to those specified in the preceding results. Assumption 3, i.e., $\rho \geq \left(\frac{1-\pi_0}{N+1}\right)^2$, streamlines the results, as shown below. We may re-write the regions within the parameter space for which each case holds as below and ask how the bounds of the intervals of $(\hat{\pi} - \pi_0)$ change as ρ and N increase.

(i) A truth-telling case obtains if

$$(\hat{\pi} - \pi_0) \in \left[\frac{1 - 2\pi_0}{2(N+1)} - \frac{\rho(N+1)}{2}, \frac{1 - 2\pi_0}{2(N+1)} + \frac{\rho(N+1)}{2}\right].$$

Taking the derivative of the bounds with respect to ρ, we may easily see that the lower bound decreases and the upper bound increases, such that the larger is ρ, the greater the interval of $(\hat{\pi} - \pi_0)$ that are compatible with a truth-telling equilibrium.

The length of the interval itself is clearly increasing in N, even if not in the sense of set inclusion. Assumption 3 provides the set inclusion result. Taking the derivative of each bound with respect to N, we see the lower bound decreasing and the upper bound increasing as long as $\rho > \frac{1-2\pi_0}{(N+1)^2}$, which is implied by assumption 3.

(ii) A mixing-when-$\omega = 1$ case obtains if

$$\frac{1 - \pi_0}{2(N+1)} - \frac{\rho(N+1)}{2(1 - \pi_0)} < (\hat{\pi} - \pi_0) < \frac{1 - 2\pi_0}{2(N+1)} - \frac{\rho(N+1)}{2}.$$

Both bounds are decreasing in both ρ and N, so the interval of $(\hat{\pi} - \pi_0)$ compatible with mixing is increasing in the strong-set order as ρ and N fall.

(iii) A pooling-on-$s = 0$ case obtains if

$$(\hat{\pi} - \pi_0) < \min\left\{\frac{1-\pi_0}{2(N+1)} - \frac{\rho(N+1)}{2(1-\pi_0)}, \frac{1-2\pi_0}{2(N+1)} - \frac{\rho(N+1)}{2}\right\}.$$

The boundary of this region is the lower envelope of decreasing, affine functions of ρ. The derivative with respect to ρ of each possible upper bound is negative, such that the interval over which the equilibrium obtains is decreasing in the sense of set inclusion as ρ increases. The derivative with respect to N of each is also negative, yielding an analogous conclusion.

(iv) A mixing-when-$\omega = 0$ case obtains if

$$\rho(N+1) - \frac{\pi_0 - \frac{1}{4}}{N+1} < (\hat{\pi} - \pi_0) < \frac{\rho(N+1)}{2} + \frac{1-2\pi_0}{2(N+1)}.$$

Both bounds are increasing in ρ, such that the interval of $(\hat{\pi} - \pi_0)$ is increasing in the strong set order in ρ. Under assumption 3, we have both of the bounds also increasing in N and thus again conclude the interval is increasing in the strong set order.

(v) A pooling-when-$s = 1$ case obtains if

$$(\hat{\pi} - \pi_0) < \max\left\{\rho(N+1) - \frac{\pi_0 - \frac{1}{4}}{N+1}, \frac{\rho(N+1)}{2} + \frac{1-2\pi_0}{2(N+1)}\right\}.$$

The boundary is the upper envelope of increasing affine functions. The derivative of each possible bound with respect to ρ is unambiguously positive, such that the interval over which the equilibrium obtains is decreasing in the sense of set inclusion as ρ increases.

The derivative of the bounds with respect to N are positive if $\rho > \frac{\frac{1}{4}-\pi_0}{(N+1)^2}$ and if $\rho > \frac{1-2\pi_0}{(N+1)^2}$, both of which hold under assumption 3, such that the interval over which the equilibrium obtains is also decreasing in the sense of set inclusion as N increases.

■