Incentives or Disincentives?

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Abstract

If a policymaker wishes to encourage members of a population to take a socially beneficial action, should she reward those who take the desired behavior or punish those who do not? This paper develops a model that facilitates both utilitarian and majoritarian perspectives on the use of incentives and disincentives in public policy. An asymmetry arises between the two types of policy: as incentives get larger, a higher share of the population takes the beneficial action and thus earns the reward, driving costs up; as disincentives get larger, however, a smaller share of the population fails to take the desired action, requiring fewer fines be administered and exerting downward pressure on costs. Policy domains that favor inducing high (resp., low) shares of the population to take a desired behavior thus also indirectly favor the use of disincentives (resp., incentives). Distributive implications for members of the population only amplify this tendency, such that majoritarian influence on policymaking tends to generate stronger disincentives and weaker incentives than would be chosen by a utilitarian policy planner.

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> . . . the only fair way to begin must be with the tenet that there is no basic or universal rationale for having a general predisposition toward one control mode or the other. . . Even on an abstract level, it would be useful to know how to identify a situation where employing one mode is relatively advantageous, other things being equal.
>
> Weitzman (1974)

1 Introduction

One of the fundamental tasks facing a government is to influence the behavior of the population it governs. In the simplest setting we might consider, a government may wish to induce members of a population to take one action instead of another. Should the government's policy reward those who take the desired action or punish those who do not? How do electoral pressures affect the choice between using incentives and disincentives?

Answers to these foundational questions about the use of incentives and disincentives tend to rely on idiosyncrasies of a given context or mere path dependency for justification, rather than a more basic understanding of the difference between what might appear at first blush to be two sides of the same enforcement coin. Already an unsatisfying state of affairs, such an incomplete understanding further precludes an analysis of the role politics plays in choosing among incentives and disincentives.

Such questions are hardly idle thought experiments. In practice, policymakers turn to both incentives and disincentives, even within a given policy domain, to achieve similar aims. Amidst lagging rates of vaccination to COVID-19, for instance, policymakers have considered both incentive- and disincentive-based policies to induce individuals to protect themselves and others (Savulescu, Pugh & Wilkinson 2021). Similarly, the heart of the debate in the United States over the Affordable Care Act (ACA) was the question of whether government ought to disincentivize the failure to insure oneself or instead ought to incentivize the purchase of insurance. Central to conservatives' complaints about the ACA was the individual mandate, which requires individuals to purchase health insurance or else pay a fine. It followed, then, that a key element of all of the Republican proposals was replacing the individual mandate with tax credits intended to subsidize the purchase of insurance (RNC 2017).

In the realm of U.S. environmental policy, the government discourages polluting activities through taxes, while initiatives such as Cash for Clunkers rewarded the retirement of less efficient vehicles to subsidize the purchase of more efficient vehicles. National education policy has featured both disincentive- and incentive-based policies in recent years in the form of No Child Left Behind and Race to the Top, respectively (Howell 2004, Howell & Magazinnik 2017). The domain of agricultural and food policy abounds with examples: the

Conservation Reserve Program incentivizes farmers to take land out of agricultural production (Wilde 2013, USDA 2020), the infamous "sugary drink taxes" are a disincentive for unhealthy consumption (Neuman 2009), and a variety of programs reward individuals for healthy consumption choices. These latter programs include doubling the value of food stamps used for fruits and vegetables and providing incentives for children to choose better school lunch options (Just & Price 2013, List & Samek 2017).

This paper uses a formal model to offer normative (viz., utilitarian) and positive (viz., majoritarian) perspectives on the choice between incentive- and disincentive-based policies. Comparing the majority-preferred policy to the utilitarian benchmark characterizes how the policy outcome of a democratic process is likely to differ from the efficient policy. The setting is kept as simple as possible while still emphasizing the central features of a policymaking environment. Each member of a population chooses one of two actions, one of which is more socially beneficial than the other. The propensity to choose this action – in the absence of any policy inducements – varies across members of the population. Throughout, the model assumes incentives and disincentives influence the behavior of members of the population identically. Adopting such a stark, rational-choice perspective challenges the model to disprove the null hypothesis that incentives and disincentives are equivalent means to the same end. Section 2 situates this approach and the findings that stem from it within a number of related literatures, and Section 3 makes precise the assumptions and structure of the model.

Policies apply to populations, not single entities, so all those who take an incentivized behavior are entitled to the same size of any reward being offered, while all those who do not will receive the size of any punishment on the books. An important asymmetry then follows when combined with the premise that the administrative cost associated with a given policy grows in the share of the population receiving that policy. Increasing the size of an incentive induces a larger share of the population to take the desired action, which in turn increases the share of the population who must receive the reward, causing administrative costs to rise. Increasing the size of a disincentive also induces a larger share of the population to take the desired action, but this reduces the share of the population who must receive the punishment, causing administrative costs to fall. As such, the administrative cost of disincentives (resp., incentives) falls (resp., rises) as a larger share of the population takes the desired action.

With the key difference between punishment and reward policies being the way in which their costs depend on the share of the population complying, both the type of policy (incentives or disincentives?) as well as the size of the intervention (how big is the reward/punishment, how much compliance does it induce?) are of interest. Inducing higher shares of the population to take the beneficial behavior and the use of disincentives (rather than incentives) are complements to one another. Section 4 explores the consequences of this complementarity for the type and size of intervention a utilitarian social planner would endorse.

Broadly, conditions that favor inducing higher (resp., lower) shares of the population to take the beneficial action indirectly favor the use of disincentives (resp., incentives). A higher marginal societal benefit associated with the desirable action, for example, indirectly favors the use of disincentives relative to incentives.

The complementarity between inducing higher shares of the population to take the beneficial action/avoid a harmful action and the use of disincentives (rather than incentives) grows even stronger in the context of the majority preference than it was under a utilitarian perspective. While a utilitarian perspective is neutral towards redistributive transfers, members of the population are not transfer neutral. The larger the incentive or disincentive, the more likely the policy is to induce any given member of the population to take the desired behavior. Furthermore, if a member of the population takes the beneficial action, the redistributive implications lead her to prefer a larger policy intervention. A larger intervention, though, induces a larger share of the population to take the beneficial action, which in turn makes disincentives relatively more cost effective than incentives. Section 5 analyzes the preferences of members of the population and characterizes the majority voting equilibrium.

Intuition may have suggested that individuals would always prefer incentive-based policies. Because of the redistributive implications of policies, however, individuals may well favor disincentive-based policies. Section 5 demonstrates that a majority of the population will tend to prefer either larger-than-optimal disincentives or smaller-than-optimal incentives. Furthermore, while a utilitarian policymaker would never seek to increase the share of the population choosing the less beneficial (more harmful) action, this need not be the case under the majority-preferred policy. Indeed, policies that encourage the harmful behavior, usually through a tax on the beneficial behavior, arise in the analysis of majority-preference as an example of insufficiently large incentive policies. The majority-preferred incentive for the desirable behavior may be so small as to be negative, actually constituting a fine or tax on the socially-beneficial behavior. The suggestion that policies may run counter to societal benefit is perhaps not so surprising. Interestingly, policies encouraging and discouraging the beneficial behavior may have highly similar implications for the utility of the median voter, despite their drastically different intentions. Section 6 summarizes several extensions, such as allowing for potentially imperfect enforcement and considering policies that target a sub-population of interest. Section 7 briefly concludes.

2 Literature Review

Two principles receive priority below in modeling the use of incentives and disincentives as policy instruments. The first, per the sentiment from Weitzman quoted at the top of the paper, is to set up the two types of policies to be as similar as possible, without assuming they are necessarily interchangeable from a policymaking perspective.[1] The second principle guiding the modeling choices below is to embody the core realities of policymaking, namely that policies apply to populations (not single individuals) that consist of members with heterogenous propensities to comply with desired behaviors. When these two tenets are at odds, the former receives priority.

In work extending back to Stigler (1971), Posner (1974), and Peltzman (1989), economists have studied the manipulability of regulatory environments, paying less attention to the particular choice of policy. Policy choice has been taken up in more specific program evaluation and experimental contexts, while political scientists have focused on the manipulability of stages of the policymaking process surrounding the choice of policy instrument. The large literature on bureaucracy, for example, has devoted significant attention to the decision of a political principal to delegate decisions, such as those over the policy instruments, to other actors within the government (Gailmard & Patty 2013). In general, the delegation literature is premised on a conflict between a politically-sensitive principal and a public service-motivated bureaucrat in the sense of Weber (1948, §VIII) or Miller & Whitford (2016). Yet the way in which utilitarian and majoritarian perspectives may be at odds, especially with reference to the choice of policy instrument, remains opaque. Even when it comes to the implementation of policy decisions, as in Pressman & Wildavsky's (1984) landmark study, the focus is on the target of policies, rather than the choice of instrument with which to pursue a given target. This paper provides an account of the source and nature of the conflict that may emerge between governmental actors with more utilitarian and those with more majoritarian perspectives.

Standing in opposition to the work that treats incentive and disincentive policies as entirely interchangeable, public administration and legal scholars exploring alternative approaches to regulation have explicitly weighed "punishment" against "persuasion" (see Gunningham (2012), Baldwin, Cave & Lodge (2012, ch. 7), Lodge & Wegrich (2012, pp. 76-80), and De Geest & Dari-Mattiacci (2013)). While they highlight a variety of potential asymmetries between the regulatory strategies of incentives and disincentives, it is the embrace of these dissimilarities from the outset that prevents this work from speaking to more fundamental differences. To serve as an effective counterpoint to the notion that incentives and disincentives constitute essentially the same enforcement technology, one must begin more agnostically, treating the two approaches identically. Meanwhile, public enforcement work tends towards the study of punishment (Polinsky & Shavell 2000). Becker (1968) is an exception, considering rewards as well as punishments, though even he treats the decision to employ one control mode or the other as a settled matter.

[1]Weitzman wrote on price vs. quantity policies; both incentives and disincentives constitute variants of price policies. The difference between price and quantity policies in his analysis and follow-on papers hinges on uncertainty about the benefits and costs of regulation (Pizer 1997, Grodecka & Kuralbayeva 2015), while the model herein relies only on the heterogeneity of actors within a population.

A sizable body of behavioral work has studied differences likely to arise in individual-level responses to positive and negative inducements (e.g., Kaplow & Shavell 2007, Benabou & Tirole 2011). In seeking to remain agnostic about the differences between incentives and disincentives, behavioral differences such as these are set aside below. Members of the population will respond identically to incentives and disincentives of the same magnitude. In contrast to the behavioral work, studies imposing more strict rationality on actors have traditionally, if implicitly, assumed incentive and disincentive policies are equivalent enforcement mechanisms without further investigation. The supposed equivalence is reminiscent of the symmetry in assigning liability in accidents/externalities in scholarship on law, politics, and economics (Coase 1960, Calabresi & Melamed 1972, Miceli 2004, Posner 2005). In broad brush strokes, a variety of disciplines within the social sciences have taken as given the equivalence of positive and negative inducements in policy settings when studying public policy.

As stated, a central tenet of the model below is that policies apply to populations of heterogeneous individuals. Moral hazard models in the tradition of Holmstrom (1979) tend to feature only one agent (Banks & Sundaram 1998, p. 299) or a collection of homogeneous agents (with some exceptions (Dubois & Vukina 2009)). Limiting investigation of a policy's effect to a single actor, as in Dal Bo, Dal Bo & Di Tella (2006), means both types of policy may be "on the books" with only one deployed in equilibrium. This makes the choice of instrument less consequential than it is in the context of a population of varied actors. Policies applied to a population will almost always result in compliers and non-compliers. If both incentives and disincentives were on the books, both types of policies would need to be deployed in equilibrium.

Distributive conflict animates the analysis below of the majority-preferred policy, though the conflict differs in its origin from much prior work on redistribution. The field of distributive politics often takes redistribution itself as the primary source of distributive conflict (Baron & Ferejohn 1989, Becker 1983, Becker 1985). To the extent that other policy domains appear, it is often as an entirely separate dimension from a policy of interest, as in the literature on vote buying (Snyder, Jr. 1991, Groseclose & Snyder, Jr. 1996, Dekel, Jackson & Wolinsky 2008, Dekel, Jackson & Wolinsky 2009). The distributive conflict featured here, however, arises as a byproduct of policymaking in domains that are not redistributive in nature.

The distributive implications of policies in fact lead to a somewhat different set of predictions than those in Olson's (1965) classic treatise discussing collective-action dilemmas. Olson suggests that when benefits are diffuse and costs concentrated, societies will likely underprovide beneficial actions/overprovide harmful actions. Olson then considers potential solutions, such as coerced participation (via disincentives) or selective incentives. Olson does not consider the choice between incentives or disincentives, nor does he model the ways in which administrative costs affect the desirability of various solutions to collective action dilemmas or the implications that transfers have on the popularity of such potential solutions. The analysis of the majority-preferred policy below demonstrates that (popular) solutions to collective action problems may still result in suboptimal outcomes, differing from the suboptimal *ex ante* conditions in predictable ways. The model herein predicts a majority of the population may prefer larger-than-optimal punishments that result in an over provision of beneficial behavior among the population. It also predicts that a majority may favor smaller-than-optimal rewards that lead to continued under provision of beneficial behavior, only partially ameliorating the suboptimal *ex ante* conditions. The emphasis on comparing possible solutions to collective action dilemmas places the present paper more in the tradition that followed Olson's seminal work, such as Ostrom's work on institutional choice in the face of challenges around common-pool resources (Ostrom 1990).

3 A Model of Incentives and Disincentives

The model considers a population consisting of a unit mass of individuals. Denote an arbitrary member of the population by i. All members simultaneously choose $a_i \in \{0, 1\}$ exactly once.

Let v_i represent i's *ex ante* valuation for choosing $a_i = 1$ (i.e., in the absence of any incentives or disincentives), with $\underline{v} \leq v_i \leq \overline{v}, \forall i$.[2] Note that v_i may be negative, indicating a latent propensity to take $a = 0$, or positive, indicating that i would choose $a_i = 1$ without any further inducement. Indeed, assume $\underline{v} < 0 < \overline{v}$, such that there are *ex ante* compliers and non-compliers in the population. Let the share of the population with valuations below some level v be given by a continuously differentiable function $F : [\underline{v}, \overline{v}] \longrightarrow [0, 1]$.

When members of the population choose $a_i = 1$ instead of $a_i = 0$, it imparts social benefit (or, equivalently, avoids social harm). Denote the share of the population choosing $a = 1$ by Γ. Let the function $W : [0, 1] \to \mathbb{R}$ represent the external benefit to society from a given share of the population, Γ, choosing the desirable action, $a = 1$, and let it be continuously differentiable, with $W' > 0$. It is assumed that all externalities of individual choices are represented by W. This, along with the measurability of W with respect to the share of the population choosing $a_i = 1$, implies that all citizens' contribution decisions are equally weighted from a social planner's perspective, even though individuals face heterogeneous individual net costs of contributing as captured by the v_i.

A policymaker wishing to encourage citizens to take action $a = 1$ rather than $a = 0$ may reward those who take the beneficial action ($a = 1$), punish those who do not ($a = 0$), both, or neither. Assume all members of the population are subject to the policy.[3] To capture the constraints of policymaking, assume

[2]This term captures the net individual benefit of taking $a_i = 1$ instead of $a_i = 0$.

[3]An extension in Appendix C considers the presence of an "unaffected" subpopulation – members of the population not subject to a choice of $a \in \{0, 1\}$ or the corresponding rewards or punishments yet still affected

that the policy cannot impose different levels of reward or punishment across the population. This requires that any individual in the population receiving a reward receives the same level of reward, and similarly for punishments. Further, incentives, R, and disincentives, P, enter additively into the utility functions of individuals as perfect substitutes.[4] Suppose further that the policymaker has full information, sidestepping issues of imperfect enforcement.[5]

Total rewards are given by $R \cdot \Gamma$, as the entire share of those choosing $a = 1$ must be given the reward, which is of size R. This represents an addition to the utility of those receiving the reward but must be financed through taxation. Total punishments equal $P \cdot (1 - \Gamma)$, as the share of those choosing $a = 0$ must receive the punishment. This represents a subtraction from the utility of those receiving the punishment but is then redistributed to the population as a whole. Lump-sum transfers applied uniformly across the population accomplish both the disbursement of revenues from fines as well as the financing of subsidies, à la Meltzer & Richard (1981). All members of the population thus receive an equal share of revenues collected from fines and carry an equal burden in financing subsidies. Given disincentives, P, and incentives, R, and a share of the population choosing $a = 1$, Γ, an individual choosing a_i receives $(a_i - \Gamma) \cdot (P + R)$.[6]

Transfers are costly to administer, however, adding to the cost of financing rewards and reducing the revenue from fines available for redistribution. Let $C_p : [0, 1] \to \mathbb{R}$ and $C_r : [0, 1] \to \mathbb{R}$ be differentiable functions representing the administrative cost incurred to apply disincentives and incentives, respectively, to a given share of the population. As such, C_p takes as its argument the share of the population choosing $a = 0$, $1 - \Gamma$, while C_r takes as its argument the share of the population choosing $a = 1$, Γ. Let $C_p(0) \geq 0$ and $C_r(0) \geq 0$, where strict inequalities at 0 imply non-zero fixed cost. To capture the notion that the costs of administering a policy are increasing in the measure of individuals to whom the policy is applied (those receiving rewards and/or those receiving punishments), $C'_p(\cdot) > 0$ and $C'_r(\cdot) > 0$.

The utility formulation in equation (1) represents the preferences of a given member of the population, i, as a function of their action (a_i), given their valuation (v_i), incentives (R), disincentives (P), and a share of

by redistribution.

[4]Receiving R will always be contingent on $a = 1$, paying P will always be contingent on $a = 0$; the sequel considers the possibility of majoritarian support for $-R$, a fine for those choosing $a = 1$, and $-P$, a reward for those choosing $a = 0$.

[5]Appendix B incorporates two potential sources of incomplete information.

[6]An *ex post* noncomplier funds subsidies, incurring $-\Gamma \cdot R$, and/or receives a fine net of redistribution of that fine, i.e., $-P + (1 - \Gamma) \cdot P = -\Gamma \cdot P$. An *ex post* complier receives the reward net of funding the subsidy, i.e., $R - \Gamma \cdot R = (1 - \Gamma) \cdot R$, and/or receives only the revenue extracted from noncompliers but does not incur a fine, $(1 - \Gamma) \cdot P$.

the population (Γ) choosing $a = 1$. Since the administrative costs of a policy accrue only if the policy entails a nonzero amount of punishment or reward, let $\pi = 1$ if and only if $P \neq 0$, 0 otherwise, and let $\rho = 1$ if and only if $R \neq 0$, 0 otherwise.

$$u_i(a_i; P, R, \Gamma, v_i) = W(\Gamma) + (a_i - \Gamma) \cdot (R + P) - \rho \cdot C_r(\Gamma) - \pi \cdot C_p(1 - \Gamma) + a_i \cdot v_i \tag{1}$$

Examining the optimal choice of a member of the population and assuming i chooses $a_i = 1$ when indifferent, it follows immediately that i chooses $a = 1$ if and only if the utility she derives from doing so plus any rewards offered for choosing $a = 1$ is at least as large as the utility she receives from choosing $a = 0$ minus any threatened punishments, i.e., $v_i + R \geq -P$. The following result derives the share of the population choosing $a = 1$ given disincentives of size P and incentives of size R.[7] All proofs may be found in the appendix.

Lemma 1. *Given incentive and disincentive policies of R and P, the share of the population choosing $a = 1$ is $1 - F(-R - P)$, while the share of the population choosing $a = 0$ is $F(-R - P)$.*

Lemma 1 permits a rewriting of Γ as $1 - F(-R - P)$ and of i's utility as $u_i(a_i; P, R, v_i)$. In practice, however, we will move to a formulation that treats the share of the population choosing $a = 1$ as a choice variable. Before proceeding further in the analysis of optimal policies and policies that would receive majoritarian support, the next subsection offers some interpretation and discussion of the model's structure.

3.1 Comments on the Model

The incentives and disincentives in the model need not be strictly pecuniary, but they must possess the redistributive implications laid out above. This precludes the analysis of punishments without a redistributive element, e.g., prison sentences, which do not offer society anything akin to the redistribution of collected fines but rather require significant public financing. In addition to subsidies and fines, however, the model may accommodate incentives that provide in-kind rewards or disincentives that require community service.

The cost functions described above represent assignment, collection, and disbursement costs specific to each type of policy and incurred only if that given type of policy is in use. These costs are variable. They accrue monotonically in the share of the population to which they must be applied. Consider the only

[7]While a member of the population's optimal action does not depend on the share of the population choosing $a = 1$ in this model, Γ is nonetheless an equilibrium quantity. This statement thus tacitly imposes a perfection requirement, in that all actors in the model consider how citizens will respond after every (P, R) pair.

somewhat stylized case of a community official tasked with preventing residential food waste (Radil 2015). The official must stop at each house to verify food has not been disposed of with the trash that heads to the landfill instead of in designated compost bins. Under a disincentive scheme, the official must spend additional time at each house that does not comply to write tickets; the cost of this policy thus rises in non-compliance. Under an incentive scheme, the official must spend additional time at each house that has properly composted its food waste to fill out a reward or subsidy (perhaps a voucher for the bins or for compostable bags); the cost of this policy thus rises in compliance.

Certainly, the two types of policy often share many costs in common. In many instances, monitoring costs are incurred across the entire population and would be necessary for either type of policy. Such costs are thus fixed costs. In the case of the Affordable Care Act, all tax returns require monitoring, albeit largely automated. Indeed, even the assignment of a subsidy or fine would be automated. In what way, then, would administrative costs differ or even be relevant? The collection and disbursement of fines and subsidies entail non-trivial deadweight administrative costs that accrue asymmetrically, and the functions specified above capture these asymmetric variable costs.

The differences between incentives and disincentives in the model this paper presents arise from the fundamental asymmetry in the cost functions for incentive and disincentive policies. In any particular policy setting, the normative or political appeal of incentives and disincentives may differ along a number of different dimensions. In nearly any such setting, however, administrative costs will accrue asymmetrically under incentives and disincentives. It is in this sense that the asymmetry in the cost functions of incentive and disincentive policies is fundamental. And it is this asymmetry in the way costs accrue under incentives and disincentives – and the implications for optimal and democratic policy choice – that previous work has not fully considered.

The analyses below seek to illustrate how the asymmetry in administrative costs – not an expressly political phenomenon in and of itself – may explain institutional variation across and within policy domains, especially emerging from democratic decisionmaking. Intuitively, it may seem incentives would always be the politically popular option. The model predicts, however, that disincentive policies may arise democratically and, when they do, that they will tend to be larger than optimal.

It is likely that administrative costs also increase in the size of the reward and/or punishment as well as the share of the population to which they are applied. The formulation above provides the most straightforward analytical approach to drawing out the central insights of the model, so the analysis proceeds as though administrative costs of a policy depend only on the measure of individuals to whom that policy is applied. Given these assumptions, however, it emerges rather starkly in Lemma 2 that it is never optimal to employ strictly positive levels of both punishments and rewards to achieve a given level of compliance across the population. The concluding section of this paper returns to the likely scenario in which administrative costs increase in the size of the intervention (i.e., R, P) as well as in the share of the population to which they are applied (i.e., $\Gamma, 1-\Gamma$). As the intervening analysis will make clear, the more general condition under which the results below would continue to hold is simply complementarity between relying on punishments more than rewards and achieving high levels of socially-beneficial behavior.

Finally, as presented above, the model does not game-theoretically ground individual actions within the population or embed them in the larger game that includes policymakers. As noted, the calculation of the share of the population choosing each action tacitly imposes a perfection requirement, and, if the population were finite, individual behavior in the event of indifference would be dictated by the requirements of subgame perfect equilibrium rather than simply assumed. More explicit foundations for the information structure and beliefs of members of the population might be necessary for extensions that considered asymmetric information or externalities among individuals.

4 The Utilitarian-Optimal Policy

A utilitarian social planner seeks to maximize total welfare, giving equal weight to the utility of all members of the population. The analysis of the utilitarian's optimal policy, (P^*, R^*), thus takes into account the following implications of a policy intervention: positive externalities from members of the population choosing $a = 1$ and/or negative externalities from those choosing $a = 0$ (both captured by $W(\cdot)$), the utility that members of the population derive from their chosen action, and the deadweight costs of administering policies. Aside from the deadweight costs, transfers have no net effect on the social planner's utility as the amount collected is equal to the amount disbursed, whether the policy entails a fine or a subsidy.

A social planner would only consider policies that encourage $a = 1$.[8] Given a lower limit ($\underline{v}$) on the valuations of members of the population for choosing $a = 1$, any policy (P, R) such that $P + R > -\underline{v}$ would be weakly dominated, so suppose no policymaker ever deploys such a policy. As such, the social planner chooses $(P, R) \geq 0$ such that $0 \leq P + R \leq -\underline{v}$.

Recalling that the costs of a given policy only accrue if that policy instrument is in use, let $\pi = 1$ if and only if $P > 0$, 0 otherwise, and let $\rho = 1$ if and only if $R > 0$, 0 otherwise. The social planner thus faces the

[8]In principle, however, a policy may encourage either $a = 1$ or $a = 0$, and indeed redistributive implications may lead to majority support for a policy of incentives for $a = 0$ or disincentives for $a = 1$.

following problem:

$$\max_{\substack{(P,R)\geq 0 \ s.t. \\ (P+R)\in[0,-\underline{v}]}} \int_{\underline{v}}^{\overline{v}} u_i(a_i; R, P, \Gamma, v_i) f(v_i) \mathrm{d}v_i$$

Given our knowledge of the optimal a_i across the population and the implications for overall compliance with $a = 1$, we may rewrite the social planner's maximization problem as:

$$\max_{\substack{P,R\geq 0, \\ (P+R)\in[0,-\underline{v}]}} W(1-F(-R-P)) + \int_{-R-P}^{\overline{v}} vf(v)\mathrm{d}v - \rho \cdot C_r(1-F(-R-P)) - \pi \cdot C_p(F(-R-P)). \tag{2}$$

The internalized effects of individuals' actions on their utility across the population are given by the sum of the valuations of all those who choosing $a = 1$. As our population is a continuum, this is given by $\int_{-R-P}^{\bar{v}} vf(v)\mathrm{d}v$, where $\{i|v_i \in [-R-P, 0)\}$ constitutes the set of those choosing $a = 1$ *ex post* that would not have chosen $a = 1$ *ex ante*, without the inducement of any incentives or disincentives. Recall that v_i accounts for the benefits and costs of choosing $a = 1$ to members of the population. As such, foregone utility or profit, or the cost of adopting a new technology, would be included in v_i. This quantity is decreasing in compliance, exerting downward pressure on the social planner's utility as more individuals switch from their *ex ante* preferred choice of behavior.

Simplifying the analysis of the optimal active policy intervention significantly, Lemma 2 establishes that the policymaker will use only punishments or only rewards, if she intervenes at all.

Lemma 2. *It is never optimal to use strictly positive levels of both punishments and rewards.*

Higher shares of the population choosing $a = 1$ increase the administrative cost of rewards but lower the administrative costs of punishment policies. It follows then that there exists a threshold share of the population choosing $a = 1$ above which punishment policies are cheaper and below which reward policies are cheaper. When indifferent, the policymaker would still never want to use both types of policy to induce that share of the population to choose $a = 1$, as this would "double up" on the costs.

With Lemma 2 in hand, the social planner's problem may be restated as follows: as though restricted to using only rewards, determine the optimal size of subsidy, R^*; as though restricted to using only punishments, determine the optimal size of fine, P^*; having found the optimal size of each type of policy, compare social welfare under P^* and R^* to find the optimal type (and size) of policy.

Results from the theory of monotone comparative statics help characterize the social planner's most-preferred policy. These techniques enable a high degree of generality, and they obviate the need for several (often opaque) technical procedures, such as the use of functional forms, checking second-order conditions, or even the invocation of various assumptions to guarantee uniqueness (Ashworth & Bueno de Mesquita 2006). In particular, there is no need to place onerous conditions on the distribution of valuations, F. To facilitate this approach, the share of the population choosing $a = 1$ *ex post*, Γ, is taken to be one choice variable, and the type of policy is taken to be the other ($\tau \in \{n, p, r\}$, where n signifies no policy intervention, p signifies the use of punishments, and r signifies the use of rewards).[9] Policies are thus given by (Γ, τ). Let Γ_0 refer to the status quo level of compliance, $1 - F(0)$, which is the only level of compliance achievable if $\tau = n$.

In practice, it is analytically helpful and more interesting to focus on active policy interventions ($\tau \in \{p, r\}$), and the formulation of policy in terms of Γ and τ facilitates this intermediate optimization problem. The decision to deploy an active policy intervention or to remain at the status quo share of the population choosing $a = 1$ ($\tau = n$) is then subsequent decision in which the policymaker's utility under an optimal policy intervention is compared to her utility under the status quo. In first considering only $\tau \in \{p, r\}$, Lemma 2 permits replacing π in equation (1) with an indicator variable θ and replacing ρ with $1 - \theta$. The optimal share of the population choosing $a = 1$, Γ^*, when restricted to $\tau \in \{p, r\}$ may not be the same as in the unrestricted problem when τ may take any value in $\{n, p, r\}$. Nonetheless, we abuse notation and write optimal active policy interventions as (Γ^*, θ^*), with the understanding that this entails finding the optimal Γ when constrained to active police interventions.

Definition 1 (Optimal Policy Intervention)**.** *The utilitarian policymaker's most-preferred policy interventions, each characterized by a pair specifying of an optimal share of the population* $\Gamma \in [\Gamma_0, 1]$ *choosing* $a = 1$ *and an optimal type of policy as indicated by* θ *(where* $\theta = 1$ *if* $\tau = p$*, and* $\theta = 0$ *if* $\tau = r$*). Let* (Γ^*, θ^*) *denote the set of such pairs, such that*

$$(\Gamma^*, \theta^*) = \underset{\theta \in \{0,1\}, \Gamma \in [\Gamma_0, 1]}{\arg\max} \; W(\Gamma) + \int_0^{\Gamma} F^{-1}(1 - \hat{\Gamma}) d\hat{\Gamma} - \theta \cdot C_p(1 - \Gamma) - (1 - \theta) \cdot C_r(\Gamma). \tag{3}$$

The social planner still has a two-dimensional choice in this reformulated problem. The choice over the type of policy (θ) compares the attractiveness of using punishments to the attractiveness of using rewards to induce a given share of the population to choose $a = 1$. The choice over the *ex post* share of the population choosing the socially beneficial action (Γ) holds the type of policy fixed. Indeed, the share of the population choosing $a = 1$ is a more meaningful measure of the magnitude of a policy intervention than the value of P or R that induces that share of compliance.[10]

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[9] The equivalence of this approach is established in Lemma 5 in Appendix A.

[10] The set of optimal policy interventions is non-empty, as the objective function is continuous and $\{0, 1\} \times [\Gamma_0, 1]$ is compact. An optimal policy intervention may then be compared to a policy of (Γ_0, n), offering utility of $W(\Gamma_0) + \int_0^{\Gamma_0} F^{-1}(1 - \hat{\Gamma}) d\hat{\Gamma}$, to determine whether a policy intervention is optimal at all.

While the domain of the maximand in equation (3) includes $(\Gamma_0, 1)$ and $(\Gamma_0, 0)$, corresponding to the use of disincentives or incentives to achieve status quo-level compliance, such a policy intervention would always be dominated by (Γ_0, n). Such policies, in which the deadweight cost of a reward or fine administered to a Γ_0 share of the population, are not without an empirical analogue. In fact, a fine of zero dollars was precisely the crux of a 2020 challenge to the Affordable Care Act (Radil 2020). A fine or subsidy of zero would never be optimal, as – at least in this model – it accrues administrative costs with no change in behavior, but, as noted, it is helpful for analytical reasons to consider this option as a possible policy intervention.

In order to understand how the optimal choices of type and size of policy respond to changes in the exogenously given elements of the model, the first step is to characterize the relationship between the two endogenous variables themselves. In the present context, the share of the population choosing $a = 1$ *ex post* and the use of disincentives (as opposed to incentives) are complementary to one another, displaying increasing differences. This complementarity may take two forms. The marginal return from an increase in the share of the population choosing $a = 1$ *ex post* (i.e., $1 - F$) is larger when using disincentives than when using incentives. Equivalently, the incremental return of using disincentives instead of incentives is larger as the share choosing $a = 1$ *ex post* increases.

Given two choice variables that display increasing differences, Γ and θ, an exogenous change which leads to an increase (resp., decrease) in the optimal value of one of the choice variables will indirectly lead to an increase (resp., decrease) in the other. Noting that all increases/decreases are weak, we may adapt the preceding statement as follows: an exogenous change that increased the optimal share choosing $a = 1$ would indirectly make the use of punishments increasingly attractive relative to the use of rewards (though rewards may still be the optimal policy). An exogenous change that made punishments more attractive relative to rewards would indirectly lead to a weak increase in the optimal share of the population choosing $a = 1$.

If the direct effect of an increase in an exogenous parameter entails an increase in all of an objective function's endogenous arguments, and if those endogenous arguments display increasing differences with one another, then any indirect effects will only serve to reinforce the direct effects. If, however, a change in an exogenous variable leads to an increase in one of the choice variables but a decrease in the other, it is impossible to characterize the overall effect of a change in the variable on the optimal policy without additional assumptions. The implication of such a finding is that a comparative static derived from a given choice of functional form would not be robust to the choice of functional forms from among those that satisfy the broad conditions stated above.

Proposition 1 regards the externality at the heart of the social planner's problem. Recall that $W(\cdot)$ denotes either the societal benefit that accrues from a share of the population choosing $a = 1$ and/or the social cost from members of the population choosing $a = 0$.

Proposition 1. *Consider* $W, \hat{W}$*, and suppose* $\hat{W}' > W'$ *pointwise. Then* (Γ^*, θ^*) *is weakly larger under* $\hat{W}$ *than under* W*.*

When the marginal benefit of compliance increases, the share choosing $a = 1$ in the optimal policy intervention increases. While there is no direct effect on the optimal type of policy to employ, an increase in the optimal share choosing $a = 1$ indirectly makes the use of disincentives increasingly attractive relative to the use of incentives. Finally, while the social planner's utility from the status quo will also increase, the social planner's utility from the optimal policy intervention increases by at least as much. Thus, it is also possible to conclude that the optimal policy intervention becomes more attractive relative to the status quo.

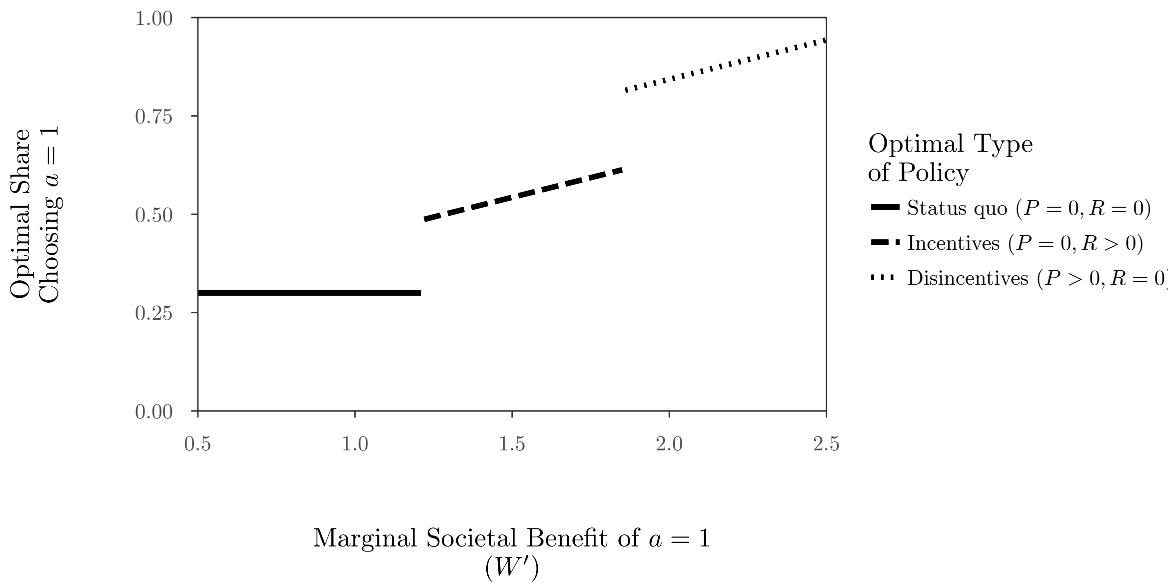
The social planner's "demand curve" for compliance shifts out because benefit from increased compliance rises for all members of the population. The distributive implications across the population, however, depend on individual members' compliance. Unlike the utilitarian, who is neutral with regards to transfers, gross of the costs of implementation, balancing the loss to some against the benefit for others, the *ex post* complier benefits from transfers and does not internalize the negative effects of transfers on non-compliers. The *ex post* complier thus has a higher marginal net benefit than the utilitarian at all levels of compliance. A member of the population choosing $a = 0$ *ex post* internalizes the redistributive losses but receives none of the redistributive gains. The *ex post* non-complier thus has a lower marginal net benefit at all levels of compliance than the utilitarian. Individual compliance thus reinforces the desire for greater population-wide compliance associated with an outward shift in W'.

Figure 1 depicts the comparative statics from Proposition 1. As the marginal societal benefit from more of the population choosing $a = 1$ increases (at all levels), the optimal share choosing $a = 1$ is weakly increasing. Accordingly, the optimal type of policy moves from rewards to punishments. Further, once an active policy intervention becomes optimal (vis-à-vis the status quo), and as long as there is no change in $W(0)$, it remains optimal to intervene as the marginal societal benefit of higher shares choosing $a = 1$ continues to increase.

It is worth noting that neither the depiction of Proposition 1 in Figure 1 nor the underlying result itself rely on the assumption of different fixed costs of the two types of policy interventions. Indeed, Figure 1 is built around functional forms for the cost functions that have no fixed costs at all. If there were no compliers with $a = 1$ (i.e., $1 - F = 0$), the cost of a reward policy would be 0. If there were full compliance with $a = 1$ (i.e., $1 - F = 1$), the cost of a punishment policy would be 0. Using incentives is preferable at lower levels of compliance precisely because lower marginal benefit to compliance leads to a lower optimal level of compliance, *ceteris paribus*, which makes rewards a more cost-effective policy relative to punishments.

Actions for which the order and well-functioning of society depend on nearly full compliance (e.g., safe driving, respecting property rights, not committing violent acts) are suggestive of a high marginal cost from

Figure 1: The effect of increasing the marginal benefit to society of compliance with $a = 1$ on the utilitarian's optimal share of the population choosing $a = 1$ and optimal type of policy



Notes: The following functional forms underlie this figure: $v \sim U(-7/2, 3/2) \Rightarrow F(-R - P) = (-R - P + 7/2)/5$, $W(1 - F) = g \cdot (1 - F)$, $C_p(F) = (5/7) \cdot F$, and $C_r(1 - F) = (2/7) \cdot (1 - F)$.
The horizontal axis tracks increases in W' (viz., g), the marginal societal benefit.

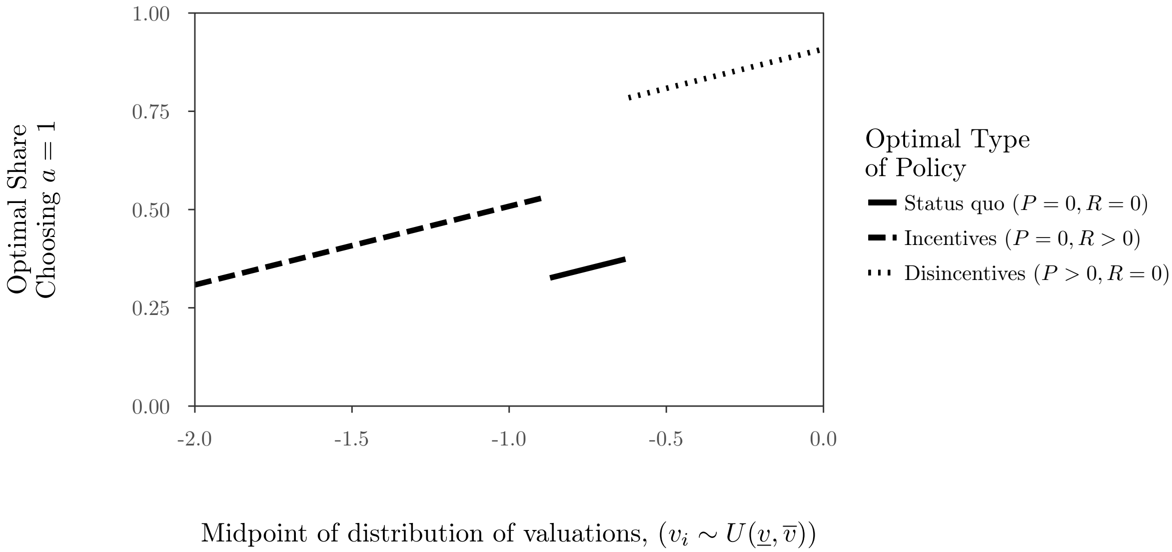
non-compliance at all levels of compliance. Achieving small levels of non-compliance with a (possibly large) punishment attain the external benefit of compliance without incurring excessive administrative costs of rewarding such behavior. As this logic suggests, societies tend to induce such actions via fines for failure to comply rather than subsidies or other incentives for compliance.

The next result considers the distribution of valuations across the population for choosing $a = 1$ rather than $a = 0$. Specifically, the statement employs an ordering akin the concept of a first-order stochastic increase. A distribution of valuations F will be said to increase to $\hat{F}$ if $\hat{F}(v) \leq F(v)$, $\forall v$, implying an increase in the share of the population that wishes to choose $a = 1$ at any given level of intervention.

Proposition 2. *Consider $F, \hat{F}$, and suppose for all $v \in (\underline{v}, \overline{v})$, $\hat{F}(v) < F(v)$. Then (Γ^*, θ^*) is weakly larger under $\hat{F}$ than under F.*

This proposition asks how the social planner's optimal policy would change if the population were *ex ante* more prone to choosing the socially-beneficial behavior, $a = 1$. In a population more prone to choosing the beneficial behavior, fewer individuals must change their behavior in order to achieve a desired share choosing $a = 1$. The direct effect is to increase the optimal share of the population choosing $a = 1$. Similar to the analysis of W, the change in the distribution of valuations, F, has no direct effect on the attractiveness of

Figure 2: The effect of increasing *ex ante* valuations for choosing $a = 1$ rather than $a = 0$ across the population on the utilitarian's optimal share of the population choosing $a = 1$ and optimal type of policy



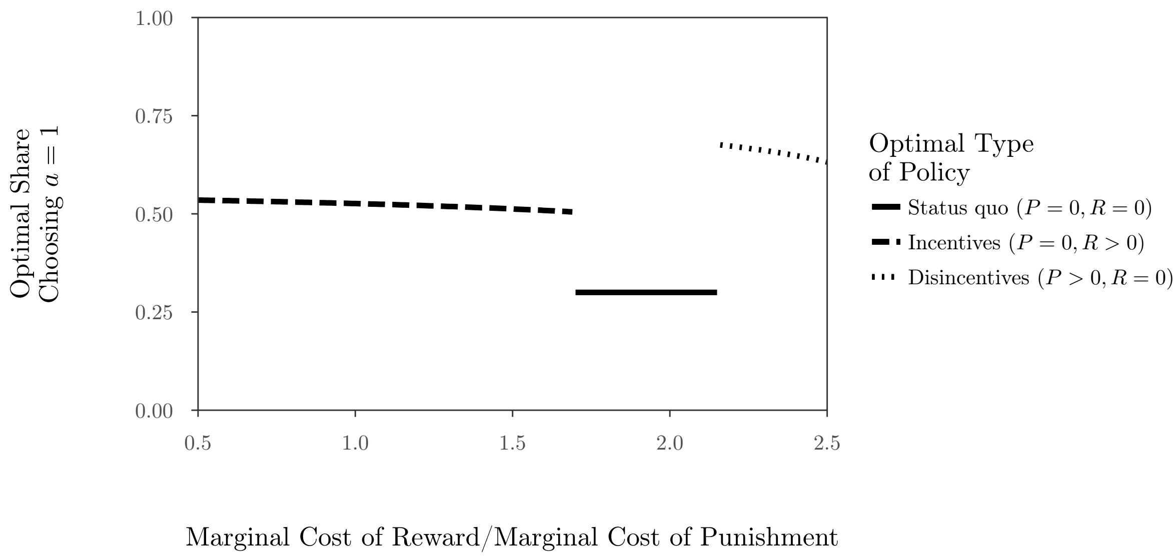
Notes: The following functional forms underlie this figure: $v \sim U(\underline{v}, \overline{v})$, with $\overline{v} - \underline{v} = 5 \Rightarrow F(-R - P) = (-R - P - \underline{v})/5$, $W(1 - F) = (11/8) \cdot (1 - F)$, $C_p(F) = (4/5) \cdot F$, and $C_r(1 - F) = (1/5) \cdot (1 - F)$.
The horizontal axis tracks $(\overline{v} + \underline{v})/2$. An increase in this quantity corresponds to an upward shift in the distribution of valuations $F(\cdot)$, a type of first-order stochastic increase.

using punishments or rewards. Through its effect on the optimal share of the population choosing $a = 1$, however, the change in F indirectly favors the use of disincentives rather than incentives.

Figure 2 illustrates these results. Moving to the right along the horizontal axis, the distribution of valuations, F, shifts upwards, a form of first-order stochastic increase. The example entails a uniform distribution of valuations, and the running variable is the midpoint of the distribution. The optimal share choosing $a = 1$ is increasing when a policy intervention is optimal, and this ultimately favors the use of punishments instead of the use of rewards. The figure also demonstrates that the optimal level of compliance and type of policy may "regress" to the status quo even as F continues to shift upwards, because the status quo is also more attractive under $\hat{F}$ than under F.

The comparative statics around the underlying propensity to take the socially beneficial behavior may speak to the different policy instruments countries use to encourage school attendance. In the United States, where public education is highly institutionalized and widely accessible – even in rural environments – disincentives (mostly fines) discourage parents from failing to enroll their children in school. In countries where public education may be less accessible and children may be expected to support their families in various capacities, incentive schemes such as conditional cash transfers seek to encourage school attendance.

Figure 3: The effect of decreasing the marginal cost of disincentives and increasing the marginal cost of incentives on the utilitarian most-preferred share of the population choosing $a = 1$ and type of policy



Notes: The following functional forms underlie this figure: $v \sim U(-7/2, 3/2) \Rightarrow F(-R-P) = (-R-P+7/2)/5$, $W(1-F) = (11/8) \cdot (1-F)$, $C_p(F) = c_p \cdot F$, and $C_r(1-F) = c_r \cdot (1-F)$.
For purposes of illustration, an increase in the ratio of marginal costs corresponds to a simultaneous increase in the marginal cost of incentives and decrease in the marginal cost of disincentives. Specifically, the horizontal axis tracks c_r/c_p, where $c_r + c_p = 1$.

Proposition 3 concerns the cost functions and is surprising precisely because of the ambiguous net effects it highlights. Adjusting the marginal costs of the types of policies would seem to be the most straightforward way to favor one type of policy over the other, but this is not the case. The changes have direct effects on both the optimal share of the population choosing $a = 1$ and the optimal type of policy, but in such ways that the indirect effects of the choice variables on one another find themselves at odds. It is impossible to state – without invoking additional assumptions – that reducing the per-unit administrative costs of a type of policy always favors the use of that policy.

Proposition 3. *Consider $C_\tau, \hat{C}_\tau$, $\tau = p, r$, and suppose $\hat{C}'_\tau < C'_\tau$ pointwise. Absent additional assumptions, (Γ^*, θ^*) may be larger or smaller under $\hat{C}_\tau$ than under C_τ.*

Lowering the marginal cost of either type of policy makes the use of that instrument less costly. The other direct effect of lowering the marginal cost of incentives is to encourage greater compliance, and the other direct effect of lowering the marginal cost of disincentives is to encourage less compliance, both of which indirectly make the other type of policy more attractive.[11] The direct effects thus lead to opposing indirect

[11] A lower marginal cost of disincentives makes it less expensive to dole out a greater number of fines via

effects. Even if reducing (resp., increasing) the cost of disincentives (resp., incentives) does not entail strong enough indirect effects so as to result in a switch to incentives (resp., disincentives), it would still lead to a lower optimal share of the population choosing $a = 1$, an unexpected outcome of making policy interventions less costly.

Figure 3 shows exactly these effects. Moving to the right along the horizontal axis, the marginal cost of disincentives falls and the marginal cost of incentives rises.[12] The optimal level of compliance decreases even as the marginal cost of a disincentive-based policy falls relative to the marginal cost of incentive-based policy. Furthermore, the competing effects that occur when marginal costs change allow the status quo to be more attractive than either type of policy intervention at intermediate levels of marginal cost.

Motivated by specific questions pertaining to the change in marginal administrative costs, it may well be possible and worthwhile to break the ambiguity just described. Incorporating imperfect enforcement into the model is interesting in its own right but also constitutes an example of a question involving the cost functions that has enough structure to make a stronger statement than is possible below. Appendix B explores the possibility of imperfect enforcement, the implication for the cost of incentive and disincentive policies, and the effects on the social planner's optimal policy interventions.

5 The Majority-Preferred Policy

In contrast to a utilitarian social planner, and especially in an electoral context, politicians may seek policies that garner the support of a majority of the population. This section characterizes the majority-preferred policy, (P^{**}, R^{**}), and its relationship to the social planner's optimal policy, (P^*, R^*). The same utility function as in equation (1) represents preferences of members of the population over policies but with R and P as additional choice variables. Denote this formulation by U_i, such that $(a_i, P, R) \mapsto U_i(a_i, P, R; v_i)$. Note that we no longer consider only $P, R \geq 0$. The quantity P continues to refer to a policy applied to those who choose $a = 0$, where negative values of P would correspond to a policy rewarding those that choose $a = 0$. Similarly, R continues to refer to a policy applied to those who choose $a = 1$, where negative values of R would correspond to a policy punishing those that choose $a = 1$. These policies would encourage socially harmful actions/discourage socially beneficial behavior. The possibility of $P, R < 0$, clearly not something the social planner would pursue, receives further attention below.

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a smaller policy intervention; greater non-compliance means fewer individuals choosing their less-preferred action, a benefit for the social planner (though, of course, there is a reduction in social benefit).

[12]The comparative static in Proposition 3 does not require changes in both marginal costs; this is merely done for illustration.

As with the social planner, no member of the population would wish to employ both types of policy to induce the same share of the population to choose $a = 1$ ($P \neq 0$ and $R \neq 0$). This insight follows from precisely the same logic as it did for the social planner in the previous section.

Lemma 3. *All members of the population prefer the use of either an incentive or a disincentive policy – but not both – to induce any given share of the population to choose $a = 1$.*

As in the manner of the analysis of the utilitarian-optimal policy, the finding that an individual member of the population's most-preferred policy entails the use of at most one instrument – rewards or punishments but not both – permits an enlightening reframing of the problem. Specifically, voters may be thought of as choosing a level of compliance among the population ($\Gamma = 1 - F \in [0, 1]$), a policy instrument with which to achieve it ($\tau \in \{n, p, r\}$), and their own *ex post* compliance ($a_i \in \{0, 1\}$). In particular, the *ex post* action of the median voter, the overall level of compliance, and the type of policy are jointly determined in finding the majority-preferred policy, as the latter two were jointly determined in finding the social planner's optimal policy. It may still be, of course, that no intervention is the most preferred policy, but similar to the analysis of the social planner's optimal policy, results focus on the popular policies that entail an intervention ($\tau \in \{p, r\}$).

Definition 2 (Individually Most-Preferred Policy Interventions)**.** *The most-preferred policy interventions by a member of the population i, characterized by a share of the population $\Gamma \in [0, 1]$ choosing $a = 1$ and a type of policy as indicated by θ (where $\theta = 1$ if $\tau = p$, and $\theta = 0$ if $\tau = r$), as well as an individual compliance decision $a_i \in \{0, 1\}$. Each member in this set constitutes a solution to the problem:*

$$\max_{a_i,\Gamma,\theta} \; W(\Gamma) + (a_i - \Gamma) \cdot (-F^{-1}(1 - \Gamma)) - (1 - \theta) \cdot C_r(\Gamma) - \theta \cdot C_p(1 - \Gamma) + a_i \cdot v_i \tag{4}$$

The three decisions are weakly complementarity to one another. The use of punishments (vis-à-vis rewards) and inducing more compliance with $a = 1$ remain complements, as they were for the utilitarian. Additionally, personal compliance and the compliance of others are complements for members of the population. For any given member of the population, choosing $a = 1$ becomes (weakly) more attractive the larger the size of the intervention, i.e., the greater is P or R. Conversely, those complying with $a = 1$, prefer higher levels of overall compliance. The complementarity between an individual choosing $a = 1$ and the population-wide share of compliance highlights that, while the utilitarian social planner is neutral towards transfers, members of the population are not. The complementarities among endogenous choice variables – the increasing differences in (Γ, θ) and (Γ, a) – have the effect of amplifying one another.

The multi-dimensionality of the policy space prohibits easy characterization of voting equilibria. Invoking an assumption that restricts the type of policy in an active intervention (given by θ) to be the optimal instrument with which to achieve a level of compliance, the policy space becomes unidimensional with Γ as the choice variable. In this context, a majority voting equilibrium exists in which the most-preferred policies of the member of the population with the median valuation for choosing $a = 1$ rather than $a = 0$ ($i = MV$, where $v_{MV} := F^{-1}(1/2)$) constitute the set of Condorcet winning policies.

Specifically, since there is an optimal type of policy with which to achieve any given level of compliance, one may write $\theta^*(\Gamma)$, which is a monotonically increasing function due to the increasing differences, to describe the optimal choice of incentives or disincentives. Similarly, one may realize that a given member of the population's optimal action is entirely determined by the size of the policy intervention, $(R + P)$, and write $a_i^*(\Gamma; v_i)$, again a monotonically increasing function due to increasing differences, to describe i's choice of action given an intervention that induces a share Γ of the population to comply. Let $(\Gamma^{**}, \theta^{**})$ then denote the median voter's set of optimal policy interventions and a_{MV}^{**} the optimal action associated with each policy intervention in this set, where $a_{MV}^{**} := a_{MV}^*(\Gamma^{**}, v_{MV})$, $\theta^{**} := \theta^*(\Gamma^{**})$, and Γ^{**} satisfies

$$\arg\max_{\Gamma \in [0,1]} \; W(\Gamma) + (a_{MV}^*(\Gamma; v_{MV}) - \Gamma) \cdot (-F^{-1}(1-\Gamma)) - (1-\theta^*(\Gamma)) \cdot C_r(\Gamma) - \theta^*(\Gamma) \cdot C_p(1-\Gamma) + a_{MV}^*(\Gamma; v_{MV}) \cdot v_{MV}. \tag{5}$$

Lemma 4. *A majority of the population will prefer the median voter's most-preferred policy intervention,* $(\Gamma^{**}, \theta^{**})$*, over any given alternative* $(\Gamma, \theta^*(\Gamma))$*, and a majority of the population will choose* a_{MV}^{**} *under the policy.*

Accordingly, we refer to $(\Gamma^{**}, \theta^{**})$ as the set of majority-preferred policy interventions, with a_{MV}^{**} indicating the action a majority of the population chooses.

As the proof demonstrates, the higher the valuation of a member of the population, the greater the desired intervention (Γ). As such, preferences satisfy the (non-strict) single-crossing condition of Gans & Smart (1996) required for the median voter's *strict* preferences to be decisive in majority elections between pairs of alternatives, $\Gamma, \hat{\Gamma} \in [0, 1]$. The median voter's most-preferred policies are thus Condorcet winners and strong candidates for majority-voting equilibria. As Lemma 4 also notes, if the median voter is complying under a given policy, so, too, will a majority of the population choose to comply. The median voter's most preferred policy and her associated optimal action gives us insight into the action most of the population is choosing under that policy.

The majority-preferred policies and the social planner's optimal policies are not the same, but they display similar comparative statics as the utilitarian-optimal policies with respect to any exogenous changes that only directly interact with Γ. For example, taking a_{MV} to be exogenous for a moment, the increasing differences between a_{MV} and Γ imply that a majority-preferred policy under which a majority chooses $a = 1$ will entail larger interventions (i.e., larger Γ) than a majority-preferred policy under which a majority chooses $a = 0$. Further, the majority-preferred interventions sandwich the social planner's optimal policy intervention first from below and then from above as compliance with $a = 1$ increases. If $a^{**}_{MV} = 0$, then $(\Gamma^{**}, \theta^{**}) > (\Gamma^{*}, \theta^{*})$, and if $a^{**}_{MV} = 1$, then $(\Gamma^{**}, \theta^{**}) < (\Gamma^{*}, \theta^{*})$. The utilitarian and majoritarian optimal policies only coincide when each prefers the status quo to their preferred policy intervention.

Proposition 4. *If the majority-preferred policy intervention does (resp., does not) induce a majority of the population to choose $a = 1$ instead of $a = 0$, then the level of overall compliance will be greater (resp., less) than in the utilitarian optimal policy intervention, regardless of the type of policy used.*[13]

This result implies that a majority will neither sufficiently punish itself for choosing $a = 0$ nor sufficiently reward a minority that chooses $a = 1$ in that the policies result in under provision of $a = 1$ relative to the socially optimal policy. Alternatively, a majority will either excessively reward itself for choosing $a = 1$ or excessively punish a minority that chooses $a = 0$, achieving a higher share of the population choosing $a = 1$ than the social planner would. Furthermore, because large policy interventions favor the use of disincentives instead of incentives, while smaller policy interventions favor the use of incentives instead of disincentives, the majority-preferred policy will "tend" towards policies applied to a minority of the population. The complementarity between using punishments and inducing high levels of compliance favor two of the outcomes just described: a majority choosing $a = 1$ and imposing a larger-than-optimal disincentive on the minority choosing $a = 0$, and a majority choosing $a = 0$ and instituting a smaller-than-optimal incentive for the minority choosing $a = 1$.

Proposition 5 explores the effect on the majority-preferred policy of increasing the marginal benefit to society of compliance with $a = 1$.

Proposition 5. *Consider $W, \hat{W}$, and suppose $\hat{W}' > W'$ pointwise. Then $(\Gamma^{**}, \theta^{**})$ and a^{**}_{MV} are weakly larger under $\hat{W}$ than under W.*

Figure 4 provides an illustration of Proposition 5. The darker, black lines correspond to the majority-preferred share of the population choosing $a = 1$ and the line types indicate the type of policy. The gray lines correspond to the social planner's optimal share choosing $a = 1$, and the line types again indicate the type of policy. Per Proposition 4, the gray lines lie above and then below the black lines, switching when the policy intervention would induce the median voter to choose $a = 1$ instead of $a = 0$. The median voter's

[13] In the event that the majority-preferred or utilitarian-optimal policy interventions constitute sets, the levels of compliance are greater/less according to the strong set order.

most-preferred policy will not achieve the social planner's most-preferred share of the population choosing $a = 1$, unless both prefer the absence of a policy intervention, the status quo, to any active intervention. The gulf between the median voter's most-preferred policy and the social planner's most-preferred policy is shaded in gray.

Figure 4 captures several of the conclusions drawn about the majority-preferred policy thus far. First, it responds to exogenous changes in much the same way as the utilitarian-optimal policy, viz., for low marginal benefit of compliance, low levels of compliance and thus rewards are most-preferred. As the marginal benefit to society of compliance with $a = 1$ increases, the majority-preferred policy entails increased overall compliance as well as a shift to punishments as the policy means. Further, the majority-preferred policy sandwiches the utilitarian-optimal policy, inducing less than optimal compliance when marginal societal benefit is low and inducing greater than optimal compliance when marginal societal benefit is high. As discussed after Proposition 4, conditions that favor more of the population choosing $a = 1$ also favor the use of punishments, the majority-preferred policy intervention will tend to use larger punishments than the social planner and smaller rewards than the social planner.[14]

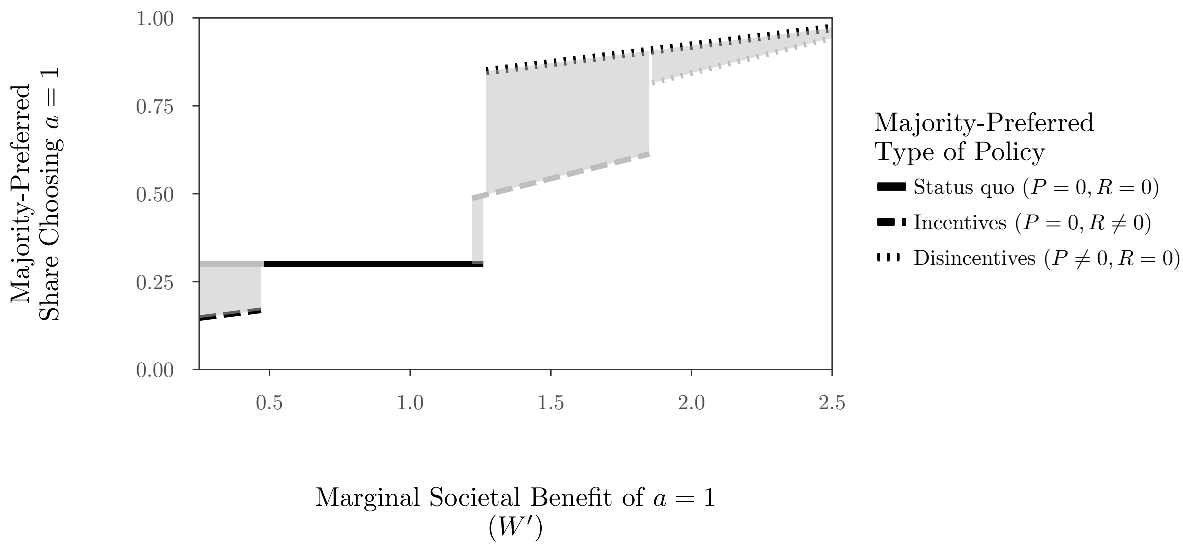
As may be further observed in Figure 4, the majority-preferred policy at low levels of marginal benefit, W', entail a "negative reward," i.e., a punishment on those taking the action $a = 1$. Such policies actually induce fewer individuals to take the socially-desirable action than would in the absence of a policy intervention. The possibility that members of the population choosing the socially beneficial behavior, $a = 1$, might be punished for doing so is a somewhat jarring implication of the results regarding the majority-preferred policy.[15] It would never have been optimal for the social planner to implement $P, R < 0$. In the context of popular support, however, encouraging the socially desirable action garners support among the population only in as much as the social benefit contributes to utility enough to outweigh redistributive concerns. The median voter need not take into account any valuations other than her own, save for the role the valuations play in determining compliance.

When the median voter chooses $a = 0$ *ex post*, the majority-preferred policy induces even smaller shares of the population to choose $a = 1$ than the utilitarian's optimal policy. The complementarity between low shares choosing $a = 1$ and the use of rewards (rather than punishments) in turn favors the use of policies applied

[14]Examples do exist of insufficiently large punishments that the majority levies on itself for choosing $a = 0$. Taxes on sugary drink purchases – often quite small – appear to be such a policy.

[15]Another (less likely) possibility is that those choosing the socially harmful behavior, $a = 0$, might be rewarded; a majority-preferred policy that deters the choice of $a = 1$, however, will likely entail $R < 0$. Since low shares of the population are choosing $a = 1$ (suggesting a choice of $a = 0$ by the median voter), it follows that a policy that applies to those choosing $a = 1$ would be the least costly.

Figure 4: The effect of increasing the marginal benefit to society of the share of the population choosing $a = 1$ on the majority-preferred share of the population choosing $a = 1$ and type of policy



Notes: The following functional forms underlie this figure: $v \sim U(-7/2, 3/2) \Rightarrow F(-R - P) = (-R - P + 7/2)/5$, $W(1 - F) = g \cdot (1 - F)$, $C_p(F) = (5/7) \cdot F$, and $C_r(1 - F) = (2/7) \cdot (1 - F)$.
The horizontal axis tracks increases in W' (viz., g), the marginal societal benefit.
The black lines correspond to the majority-preferred share of the population choosing $a = 1$, with line types indicating the type of policy. The content of Figure 1 appears shaded gray in this figure, i.e., The gray lines correspond to the utilitarian-optimal share choosing $a = 1$, with the same line types indicating the type of policy. The shaded areas are a representation of the inefficiency in the majority-preferred policy vis-à-vis the utilitarian-optimal policy.

to those choosing $a = 1$. At the extreme, these "rewards" are so small as to become fines for those choosing the socially beneficial action (see Figure 4). While such policies may seem radically different than policies encouraging the socially-beneficial behavior, it is important to note that a policy barely encouraging the beneficial behavior and a policy barely discouraging the beneficial behavior provide the median voter nearly equivalent utility. As such, conditions giving rise to a policy encouraging $a = 1$ and a policy discouraging $a = 1$ may be more similar than their stated intentions suggest.

Finally, the tools developed thus far lend themselves nicely to a more general analysis of the decisive voter. Suppose that not all of the population votes or is involved in the deliberation over the policy intervention, due to limited franchise, low turnout, lack of descriptive representation, or other democratic shortcomings.[16] Denote the median member of the voting/deliberating population, the decisive voter, as DV with valuation v_{DV} (no longer fixed at $F^{-1}(1/2)$ and action $a_{DV} \in \{0, 1\}$. The higher v_{DV}, the more electoral participation is skewed towards those with higher valuations for choosing $a = 1$ rather than $a = 0$.

[16]In contrast, Appendix C considers decision-making by a member of the population not faced with the decision of $a = 1$ and $a = 0$ and thus affected by P and R only through redistributive consequences.

Corollary 1. *Let $v_{DV} < \hat{v}_{DV}$. Then $(\Gamma^{**}, \theta^{**})$ and a^{**}_{DV} are weakly larger under $\hat{v}_{DV}$ than under v_{DV}.*

Higher valuation for *ex ante* compliance naturally makes complying *ex post* more attractive. Complying *ex post* leads the decisive voter to favor larger policy interventions that induce greater overall shares of the population to choose $a = 1$. In turn, higher levels of population-wide compliance favor the use of disincentives rather than incentives. Relative to the utilitarian optimal, these punishments will be excessive. The result above, then, suggests that the more prone the decisive voter is to comply with the socially desirable behavior *ex ante*, the more likely her most-preferred policy will entail larger-than-optimal punishments for those not choosing $a = 1$. Conversely, the less likely the decisive voter would be to choose $a = 1$ *ex ante*, the more likely her most-preferred policy would entail insufficient rewards, inducing a smaller-than-optimal share of the population to choose $a = 1$. Indeed, these rewards may be so small as to constitute a punishment for those choosing $a = 1$.

6 Extensions

The model has proven amenable to extensions, and the core insights have displayed robustness. Several such extensions appear in the appendix. This final subsection briefly describes those extensions, the more general conditions on the cost functions under which the model's insights continue to hold, and some additional directions for future work on the choice between incentive and disincentive policies.

Appendix B incorporates two sources of incomplete information. First, it considers the setting in which the median valuation is unknown by competing political parties. Under the same assumptions that reduced the dimensionality of the choice of policy above, it follows that platforms in two-candidate competition converge to the median of the distribution of medians as in Calvert (1985) and Duggan (2008). Second, it investigates the possibility of imperfect enforcement. Under reasonable additional assumptions, failure to correctly identify noncompliers (compliers) leads to an increase (decrease) in the optimal level of compliance and favors the the use of disincentives (incentives).

Appendix C supposes those facing the decision between $a = 1$ and $a = 0$ are a subpopulation and do not constitute a majority of the population. In this context, then, the median voter neither faces the decision nor the prospect of receiving any incentive or disincentive, though such an individual does have preferences over policy based on the redistributive consequences. A median voter analysis reveals the only policies now possible are those deemed above as favored, viz., larger-than-optimal disincentives or smaller-than-optimal incentives. Among other insights, this analysis suggests an explanation for "negative incentives" applied to a small subpopulation for taking socially-beneficial behavior. The fees necessary for organic certification of farms constitute one such example of this phenomenon.

Regarding the assumptions underpinning the cost functions, it is natural to wonder whether the results above hold if the administrative costs of policies increase in the size of the policy as well as the share of the population to which they are applied. Broadly, as long as the latter source of cost is a sufficient driver of overall cost, then the results continue to hold. The monotone comparative statics framework facilitates a precise statement of sufficient conditions on the cost functions that ensure similar conclusions as those derived above.

Let overall costs of any incentives and/or disincentives be described by a mapping $(\Gamma, \theta) \mapsto C(\Gamma, \theta)$, where Γ continues to represent the share of the population choosing $a = 1$, but $\theta \in [0, 1]$ represents the proportion of punishments in the overall efforts to achieve Γ. Specifically, $\Gamma = 1 - F(-X)$, $P = \theta \cdot X$, and $R = (1-\theta)X$, such that $\Gamma = 1 - F(-R - P)$. If

$$\frac{\partial^2 C}{\partial \Gamma \partial \theta} < 0, \tag{6}$$

then θ and Γ display increasing differences (costs being subtracted from objective functions above). If both variables display weakly increasing differences with an exogenous variable, then an increase in that exogenous variable will lead to a weakly larger optimal (Γ, θ). An increase in the pair implies greater compliance with $a = 1$ and greater reliance on punishments (including switches from complete reliance on rewards to complete reliance on punishments as well as the use of both policy instruments but with greater proportional reliance on punishments). This condition requires that the cost of increasing the share of the population choosing $a = 1$ (marginal cost with respect to Γ) be decreasing in the reliance on punishments rather than rewards, θ.

A more general formulation of administrative costs, such as that just sketched, enables an exploration of the model's robustness to certain behavioral effects. Suppose, for example, that loss aversion renders members of the population more sensitive to increasing fines than increasing subsidies. Greater reliance on punishments would lower the marginal cost of inducing a given share of the population to choose the socially-beneficial action. It would then be even more likely that (6) would be satisfied than under an assumption of identical responses to incentives and disincentives.

7 Conclusion

This paper began with a set of straightforward yet previously neglected questions: Should policy seeking to encourage a behavior offer incentives to those who take it or threaten disincentives to those who do not? What type of policy would a majority of the population support, and how will this differ from what a public-interested policymaker would choose? How do these answers change in response to the different conditions across policy domains?

Across both utilitarian and majoritarian perspectives, a complementarity between the use of disincentives (vis-à-vis incentives) and inducing large shares of the population to take the beneficial behavior was central to the analysis. Inducing larger shares of the population to take the beneficial action becomes more attractive as the incremental benefit to doing so increases or as the population becomes more prone to take the beneficial behavior absent any policy interventions. In turn, this favors the use of disincentives. The complementarity driving these results only grew stronger when accounting for the redistributive consequences of incentives and disincentives across members of the population. A majority will tend to prefer larger fines for those choosing the harmful action and smaller rewards for those choosing the beneficial action than a social planner would prefer. Policies that entail a disincentive for taking the beneficial action emerge as a manifestation of a smaller-than-optimal reward for those taking the beneficial action; the reward is so small as to be negative. The spirit of a policy encouraging the beneficial action and a policy discouraging it could not be more opposed, but the model demonstrates the highly similar redistributive implications of the two for the median voter, in whose eyes the policies are not so different.

Returning to the example of the Affordable Care Act (ACA), granting subsidies across the board for the purchase of insurance – including to segments of the population that required subsidies neither to pay for insurance nor to nudge them into insuring themselves – is innately inefficient. Incentives would only be optimal if most of the external benefit to society from individuals being insured accrued with only small shares of the population purchasing insurance. Insurance markets, however, are precisely the type of setting for which the shared benefit of additional participation remains high even when only a small share of the population is not participating. This feature of health policy thus favors the use of disincentives as in the mandate, so much so that popular support for the ACA was stronger than for subsidy-based alternatives.

The case of the ACA suggests a key feature of the political environment that would likely enrich future work using this model a great deal. If political parties differ systematically based on their evaluations of the externalities prompting policy intervention, and if there exists a correlation between party membership, wealth, and propensity to take the socially beneficial action, then it is almost certain that the parties will support vastly different policy interventions. Moving away from a strictly majoritarian view of the political process and incorporating party politics emerges as a particularly worthwhile direction for future study on the decision between incentives and disincentives.

Incorporating population-based behavioral effects, such as crowding in/out, norms, or coordination in interactions among the population would offer a bridge between this paper's rather stark rational choice account and the behavioral studies of the ways in which people and groups (including political parties) respond to policy interventions. Would the complementarities highlighted above persist in such settings? This remains an open question.

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Appendix

A Proofs of Results In-Text

Lemma 1. *Given incentive and disincentive policies of R and P, the share of the population choosing $a = 1$ is $1 - F(-R - P)$, while the share of the population choosing $a = 0$ is $F(-R - P)$.*

Proof of Lemma 1. Given a policy (P, R) and the assumption that i chooses 1 when indifferent, a member of the population i optimally chooses $a_i = 1$ if $v_i \geq -R - P$, and $a_i = 0$ otherwise.

The collection of members of the population choosing $a_i = 1$ are given by the set $\{i | v_i \geq -R - P\}$, representing a share given by $1 - F(-R - P)$. ■

Lemma 2. *It is never optimal to use strictly positive levels of both punishments and rewards.*

Proof of Lemma 2. Denote the share of the population with $v_i \geq 0$ by Γ_0. Recall that $C_p(\cdot)$ and $C_r(\cdot)$ are increasing in their arguments, respectively, $1-\Gamma$ and Γ. As such, there either exists a share of the population taking $a = 1$, denote it $\hat{\Gamma} \in [\Gamma_0, 1]$, such that $C_p(1-\hat{\Gamma}) = C_r(\hat{\Gamma})$, or it must be true that $C_p(1-\Gamma_0) < C_r(\Gamma_0)$ or $C_p(0) > C_r(1)$. In the latter two possibilities, the cheapest policy instruments at any share of the population choosing $a = 1$ are, respectively, disincentives and incentives.

Suppose that the first case obtains, however, where $C_p(\hat{\Gamma}) = C_r(1 - \hat{\Gamma})$ for some $\hat{\Gamma} \in [\Gamma_0, 1]$. For all lower shares taking $a = 1$, $\Gamma < \hat{\Gamma} \Rightarrow C_r(\Gamma) < C_p(1 - \Gamma)$, so rewards are the cheaper type of policy with which to attain a given level of compliance. Conversely, for higher shares of the population taking $a = 1$, $\Gamma > \hat{\Gamma} \Rightarrow C_p(1-\Gamma) < C_r(\Gamma)$, so punishments are the cheaper type of policy with which to attain a given level of compliance. At $\hat{\Gamma}$, either type of policy entails the same administrative cost, but only one should be used so that costs are not incurred twice to achieve the compliance of the $\hat{\Gamma}$ share of the population. To induce any share of the population to choose $a = 1$, then, the optimal policy will entail only one type of policy. ■

As discussed in text, we consider a choice of policy $\tau \in \{p, r, n\}$, with p denoting the use of punishments, r denoting the use of rewards, and n denoting the absence of a policy intervention, as well as a choice of the share of the population choosing $a = 1$ ex post. Lemma 5 demonstrates the equivalence of the (Γ, τ) formulation to the (P, R) formulation. In particular, it demonstrates the equivalence of the latter formulation to the approach of finding the optimal (Γ, θ) and then comparing to (Γ_0, n).

Lemma 5. *Let $X(\Gamma) = -F^{-1}(1 - \Gamma)$. If (Γ^*, τ^*) is a member of the set of optimal choices under the (Γ, τ)*

formulation, then there exists a member of the set of optimal choices under the (P, R) formulation given by

$$(P^*, R^*) = \begin{cases} (X(\Gamma^*), 0), & \tau^* = p \\ (0, X(\Gamma^*)), & \tau^* = r \\ (X(\Gamma_0), X(\Gamma_0)) = (0, 0), & \tau^* = n. \end{cases}$$

If (P^*, R^*) is a member of the set of optimal choices under the (P, R) formulation, then there exists a member of the set of optimal choices under the (Γ, τ) formulation given by

$$(\Gamma^*, \tau^*) = \begin{cases} (1 - F(-P^* - R^*), p), & P^* > 0, R^* = 0 \\ (1 - F(-P^* - R^*), r), & P^* = 0, R^* > 0 \\ (1 - F(-P^* - R^*), n), & P^* = R^* = 0. \end{cases}$$

Proof of Lemma 5. Define $X := P + R$. Given Lemma 1, there exists a one-to-one and onto relation between Γ and the value of X in equilibrium, given by $\Gamma(X) = 1 - F(-X) \Leftrightarrow X(\Gamma) = -F^{-1}(1 - \Gamma)$. Recall that Lemma 2 establishes that at most one of P or R will be strictly greater than zero. As such, when considering the optimal intervention, we may use θ to describe the type of policy in an active intervention, where $\theta = 1$ if $\tau = p$, and $\theta = 0$ if $\tau = r$. In the notation of equation (1), $\pi = 1 - \rho$ in the optimal intervention by Lemma 2, so $\theta = \pi$. As in equation (3), set

$$(\Gamma^*, \theta^*) := \underset{\Gamma \geq \Gamma_0, \theta \in \{0,1\}}{\arg\max}\ W(\Gamma) - \int_0^\Gamma X(\hat{\Gamma}) d\hat{\Gamma} - (1 - \theta) C_r(\Gamma) - \theta C_p(1 - \Gamma).$$

The main difference between the two maximization problems is the domains, so we show that the solutions to each problem lie in the domain of the other problem. By Lemma 2, the optimal policy, (P^*, R^*) lies in the set $\{(P, 0) | P \geq 0\} \cup \{(0, R) | R \geq 0\}$. By the independence of irrelevant alternatives axiom, we know that eliminating the set of policies $\{(P, R) | P > 0, R > 0\}$ and maximizing over $(P, R) \in \{(P, 0) | P \geq 0\} \cup \{(0, R) | R \geq 0\}$ yields the same solutions as maximizing over $(P, R) \in \mathbb{R}^2_+$ would yield. There exists a one-to-one and onto relation between $(P, R) \backslash (0, 0)$ and $(\Gamma, \theta) \backslash (\Gamma_0, \theta)$, where $(P, R) = (\theta \cdot X(\Gamma), (1 - \theta) \cdot X(\Gamma))$. If (P^*, R^*) maximizes equation 3 and $P^* + R^* > 0$, then $(\Gamma^*, \theta^*) = (1 - F(-R^* - P^*), \pi(P^*))$.

Note, however, that it need not be the case that (Γ^*, θ^*) as defined in (3) with $\Gamma^* > \Gamma_0$ implies $(P^*, R^*) = (\theta^* \cdot X(\Gamma^*), (1 - \theta^*) \cdot X(\Gamma^*))$, since the (Γ, θ) formulation imposes the use of rewards or punishments. This is akin to a zero-dollar fine or subsidy, in essence incurring the administrative cost without any actual transfer taking place. If $(P^*, R^*) = (0, 0)$ and $(\Gamma^*, \theta^*) = (\Gamma_0, \theta^*)$, then (Γ_0, n) will be optimal in the unconstrained formulation allowing $\tau \in \{n, p, r\}$, as it achieves the same compliance at no cost. If $(P^*, R^*) = (0, 0)$ and (Γ^*, θ^*) entails $\Gamma^* > 0$, then (Γ_0, n) will again be at least as preferred in the unconstrained problem, otherwise $(P^*, R^*) = (\theta^* \cdot X(\Gamma^*), (1-\theta^*) \cdot X(\Gamma^*))$. ■

Proposition 1. *Consider $W, \hat{W}$, and suppose $\hat{W}' > W'$ pointwise. Then (Γ^*, θ^*) is weakly larger under $\hat{W}$ than under W.*

Proof of Proposition 1. If the utilitarian objective function has monotone comparative statics in some exogenous variable z, then an increase in z will lead to an increase in the optimal policy intervention, i.e., a weakly larger share of the population choosing $a = 1$ *ex post* and either switching from using rewards to using punishments or the continued use of punishments. The definition below restates for convenience a notion of complementarity used throughout, viz., increasing differences. The next result then establishes that the utilitarian social planner's objective function displays increasing differences in the choice variables.

Definition 3 (Increasing Differences)**.** *Consider a single-valued objective function $U : Y \times Z \to \mathbb{R}$. If, for all $y, \hat{y} \in Y$ where $y < \hat{y}$ and for all $z, \hat{z} \in Z$ where $z < \hat{z}$, $U(\hat{y}, \hat{z}) - U(\hat{y}, z) > U(y, \hat{z}) - U(y, z)$, then the function U has increasing differences in the pair of arguments (y, z).*[17] *If the inequality holds weakly, refer to it as non-decreasing, and if it holds with $<$ $(\leq)$, then decreasing (non-increasing).*

Lemma 6. *The objective function in equation* (3) *has increasing differences in* (Γ, θ).

Proof of Lemma 6. Let

$$U_{SP}(\Gamma, \theta) = W(\Gamma) - \int_0^\Gamma X(\hat{\Gamma}) d\hat{\Gamma} - (1-\theta) C_r(\Gamma) - \theta C_p(1-\Gamma).$$

We show $U_{SP}(\Gamma, \theta)$ displays increasing differences in (Γ, θ) by demonstrating that $U_{SP}(\hat{\Gamma}, \hat{\theta}) - U_{SP}(\hat{\Gamma}, \theta) \geq U_{SP}(\Gamma, \hat{\theta}) - U_{SP}(\Gamma, \theta)$, for all $\hat{\Gamma} > \Gamma$ and $\hat{\theta} > \theta$ (which implies $\hat{\theta} = 1$ and $\theta = 0$).

$$U_{SP}(\hat{\Gamma}, \hat{\theta}) - U_{SP}(\hat{\Gamma}, \theta) = C_r(\hat{\Gamma}) - C_p(1-\hat{\Gamma})$$
$$U_{SP}(\Gamma, \hat{\theta}) - U_{SP}(\Gamma, \theta) = C_r(\Gamma) - C_p(1-\Gamma)$$

The first line is greater than the second if $C_p(1-\Gamma) - C_p(1-\hat{\Gamma}) \geq C_r(\Gamma) - C_r(\hat{\Gamma})$. The right-hand side is negative, as C_r is increasing in Γ. The left-hand side is positive as C_p is increasing in $1 - \Gamma$ and $\hat{\Gamma} > \Gamma \Rightarrow$

[17] Equivalently, the function U has increasing differences in the pair of arguments (y, z) if the incremental return $U(\hat{y}, \cdot) - U(y, \cdot)$ is increasing in z (or, alternatively, if the incremental return $U(\cdot, \hat{z}) - U(\cdot, z)$ is increasing in y).

$1 - \Gamma > 1 - \hat{\Gamma}$. As such, $U_{SP}(\hat{\Gamma}, 1) - U_{SP}(\Gamma, 1) \geq U_{SP}(\hat{\Gamma}, 0) - U_{SP}(\Gamma, 0)$, which was to be shown. ■

Lemma 6 provides the foundation for applying the theory of monotone comparative statics.

Definition 4 (Monotone Comparative Statics)**.** *Consider a single-valued objective function $U : Y \times Z \to \mathbb{R}$, where $z \in Z$ is an exogenous parameter. Let $Y = \times_{j=1}^{n} Y_j$, such that $(y_1, \ldots, y_n) \in Y$ is a vector of endogenous (choice) variables. If U displays non-decreasing differences in all pairs $(y_j; z)$ – with increasing differences in at least one pair – as well as non-decreasing differences in all pairs (y_j, y_k), $j \neq k$, then U displays monotone comparative statics with respect to increases in z, and an increase in z leads to a weak increase in the optimal $(y_1, \ldots, y_n)$.*[18]

Lemma 7. *Let z represent an exogenous element of the model, with a given notion of increasing for $z \in Z$. If the objective function in equation (3) has non-decreasing differences in (Γ, z) as well as in (θ, z) – with increasing differences in at least one of the pairs – then it has monotone comparative statics in z.*

Proof of Lemma 7. Letting $z = t$, $(\Gamma, \theta) = y$, $[0, 1] \times \{p, r\} = Y$, and $[\Gamma_0, 1] = S$, we recognize that the product set Y is a lattice, so we may then apply Theorem 5 from Milgrom & Shannon (1994, p. 164):[19]

> Let Y be a lattice, T a partially ordered set, and $g : Y \times T \to \mathbb{R}$. If $g(y, t)$ is supermodular in y and has increasing differences in $(y; t)$, then $\arg\max_{y \in S} g(y, t)$ is monotone nondecreasing in (t, S).[20]

Lemma 6 demonstrated increasing differences in the choice variables – satisfying the supermodularity in y in theorem quoted above. The premise of Lemma 7 supposes increasing differences between an exogenous parameter and each of the choice variables – satisfying the increasing differences in $(y; t)$ to which the quoted theorem refers.[21] ■

[18] All increases in variables are with respect to a given partial ordering. If the set of optimal $(y_1, \ldots, y_n)$ is not a singleton, then the increase in the optima is with respect to the strong set ordering.

[19] Ashworth & Bueno de Mesquita (2006, p. 221), among others, also provide statements of the conditions of complementarity under which monotone comparative statics hold without further parameterization. Specifically, and provided that the choice set is a product set, we seek to show each pair of arguments of the utility function has increasing differences. If it can then be shown that U_{SP} has increasing differences with respect to an exogenous parameter and each of the choice variables, then we may conclude that an increase in that parameter leads to an increase (in the strong set order) in the (set of) optimal of optimal policy intervention(s), (Γ^*, θ^*). We need not worry about indirect effects. The pairwise complementarity of parameters and choice variables ensures that any indirect effects only enhance the direct effects.

[20] Attributed by the authors to Topkis (1978).

[21] Relevant partial orderings on the parameter space are provided below.

It remains to be shown, then, whether the objective function in equation (3) has increasing differences in $(\Gamma, \theta; W(\cdot))$, with the partial ordering for W supplied in the Proposition. To do so, we must demonstrate increasing differences in each choice variable-parameter pair. For each parameter, we adopt a mix of techniques. To show increasing differences in θ and the parameter, we compare the incremental return of a discrete increase in the parameter under $\theta = r$ and $\theta = p$. This follows the approach taken to show the increasing differences of (Γ, θ) in the proof of Lemma 6.

We proceed differently to show increasing differences in Γ and the parameter. For example, for $W(\cdot)$, we examine $\frac{\partial}{\partial \Gamma}(U_{SP}(\Gamma, \theta; \hat{W}) - U_{SP}(\Gamma, \theta; W))$. If that quantity is positive, increasing differences may be inferred. Note that

$$\frac{\partial}{\partial \Gamma} U_{SP}(\Gamma, \theta; W(\cdot), F(\cdot), C_p(\cdot), C_r(\cdot)) = W'(\Gamma) + F^{-1}(1-\Gamma) - (1-\theta)C_r'(\Gamma) - \theta C_p'(1-\Gamma).$$

Partially order the set of functions $\{W(\cdot) | W' \geq 0\}$ with the rule:

$$\hat{W} > W \Leftrightarrow \hat{W}' > W', \forall \Gamma.$$

$(\Gamma; W): \frac{\partial}{\partial \Gamma}(U_{SP}(\Gamma, \theta; \hat{W}) - U_{SP}(\Gamma, \theta; W)) = \hat{W}'(\Gamma) - W'(\Gamma) > 0$

$(\theta; W): U_{SP}(\Gamma, \hat{\theta}; \hat{W}) - U_{SP}(\Gamma, \theta; \hat{W}) - [U_{SP}(\Gamma, \hat{\theta}; W) - U_{SP}(\Gamma, \theta; W)] = 0$

Since, U_{SP} has increasing differences in $(\Gamma; W)$ and non-decreasing differences in $(\theta; W)$, we may conclude the optimal policy intervention displays monotone comparative statics in W. ■

Proposition 2. *Consider $F, \hat{F}$, and suppose for all $v \in (\underline{v}, \overline{v})$, $\hat{F}(v) < F(v)$. Then (Γ^*, θ^*) is weakly larger under $\hat{F}$ than under F.*

Proof of Proposition 2. The proof follows the same format as the proof for Proposition 1, up to the demonstration of increasing differences between the choice variables and F.

$(\Gamma; F): \frac{\partial}{\partial \Gamma}(U_{SP}(\Gamma, \theta; \hat{F}) - U_{SP}(\Gamma, \theta; F)) = \hat{F}^{-1}(1-\Gamma) - F^{-1}(1-\Gamma) \geq 0$, ($>$ for $\Gamma \in [0,1]$)

$(\theta; F): U_{SP}(\Gamma, \hat{\theta}; \hat{F}) - U_{SP}(\Gamma, \theta; \hat{F}) - [U_{SP}(\Gamma, \hat{\theta}; F) - U_{SP}(\Gamma, \theta; F)] = 0$

Since, U_{SP} has increasing differences in $(\Gamma; F)$ and non-decreasing differences in $(\theta; F)$, we may conclude the optimal policy intervention displays monotone comparative statics in F.

Should a change in a parameter affect the constraint set for the the maximization problem, Theorem 5 from Milgrom & Shannon (1994) provides the condition under which we may still infer monotone comparative statics. Specifically, as long as the constraint set is increasing (in the strong set order) and the strict single crossing property is satisfied in the parameter (implied by the increasing differences above), we may proceed as before. Since $1 - \hat{F}(0) > 1 - F(0)$, the constraint set $[\Gamma_0, 1]$ is increasing in the strong set ordering. ■

Proposition 3. *Consider $C_\tau, \hat{C}_\tau$, $\tau = p, r$, and suppose $\hat{C}'_\tau < C'_\tau$ pointwise. Absent additional assumptions, (Γ^*, θ^*) may be larger or smaller under $\hat{C}_\tau$ than under C_τ.*

Proof of Proposition 3. The proof again follows the same format as the proof for Proposition 1, up to the demonstration of increasing differences between the choice variables and C_τ, $\tau = p, r$. Note that since we have not assumed any change to the fixed costs, $\hat{C}_r(\Gamma) < C_r(\Gamma)$ and $\hat{C}_p(1-\Gamma) < C_p(1-\Gamma)$, $\Gamma \in (0,1)$.

$$(\Gamma, C_p): \ \frac{\partial}{\partial \Gamma}(U_{SP}(\Gamma, \theta; \hat{C}_p) - U_{SP}(\Gamma, \theta; C_p)) = \theta(\hat{C}'_p(1-\Gamma) - C'_p(1-\Gamma)) < 0$$

$$(\theta, C_p): \ U_{SP}(\Gamma, 1; \hat{C}_p) - U_{SP}(\Gamma, 0; \hat{C}_p) - [U_{SP}(\Gamma, 1; C_p) - U_{SP}(\Gamma, 0; C_p)] = -\hat{C}_p(1-\Gamma) + C_p(1-\Gamma) > 0.$$

$$(\Gamma, C_r): \ \frac{\partial}{\partial \Gamma}(U_{SP}(\Gamma, \theta; \hat{C}_r) - U_{SP}(\Gamma, \theta; C_r)) = (1-\theta)(C'_r(\Gamma) - \hat{C}'_r(\Gamma)) > 0$$

$$(\theta, C_r): \ U_{SP}(\Gamma, 1; \hat{C}_r) - U_{SP}(\Gamma, 0; \hat{C}_r) - [U_{SP}(\Gamma, 1; C_r) - U_{SP}(\Gamma, 0; C_r)] = \hat{C}_r(\Gamma) - C_r(\Gamma) < 0$$

In neither the case of C_p nor C_r does the utilitarian's welfare function display increasing (or decreasing) differences between the parameter and *both* of the choice variables, which was to be shown. ■

Lemma 3. *All members of the population prefer the use of either an incentive or a disincentive policy – but not both – to achieve any given level of compliance.*

The proof follows the exact same logic as the proof of Lemma 2.

Lemma 4 *A majority of the population will prefer the median voter's most-preferred policy intervention, $(\Gamma^{**}, \theta^{**})$, over any given alternative $(\Gamma, \theta^*(\Gamma))$, and a majority of the population will choose a^{**}_{MV} under the policy.*

Proof of Lemma 4. We wish to invoke Theorem 1 from Gans & Smart (1996), which the authors summarize as stating "that when preference profiles are single-crossing, the median voters on the order of $[v \in [\underline{v}, \overline{v}]]$ are decisive in all majority elections between pairs of alternatives $[\Gamma, \hat{\Gamma} \in [0,1]]$" (p. 222, substitutions in brackets). We must demonstrate, then, that preferences over Γ are single-crossing in v. Of course, a could change as a function of changes in both v and Γ, and θ could change as a function of Γ. The latter is assumed to take its optimal value for a given Γ to reduce the dimensionality of the choice problem, as discussed in text, while sequential rationality ensures the former will take its optimal value for a given Γ and v.

Consider $v' > v$ and $\Gamma' > \Gamma$. Supposing

$$W(\Gamma') - (a_i^*(\Gamma', v) - \Gamma') \cdot F^{-1}(1-\Gamma') - (1-\theta^*(\Gamma'))C_r(\Gamma') - \theta^*(\Gamma') \cdot C_p(1-\Gamma') + a_i^*(\Gamma', v) \cdot v$$
$$\geq$$
$$W(\Gamma) - (a_i^*(\Gamma, v) - \Gamma) \cdot F^{-1}(1-\Gamma) - (1-\theta^*(\Gamma))C_r(\Gamma) - \theta^*(\Gamma) \cdot C_p(1-\Gamma) + a_i^*(\Gamma, v) \cdot v,$$

it remains to be shown that

$$W(\Gamma') - (a_i^*(\Gamma', v') - \Gamma') \cdot F^{-1}(1-\Gamma') - (1-\theta^*(\Gamma'))C_r(\Gamma') - \theta^*(\Gamma') \cdot C_p(1-\Gamma') + a_i^*(\Gamma', v') \cdot v'$$
$$\geq$$
$$W(\Gamma) - (a_i^*(\Gamma, v') - \Gamma) \cdot F^{-1}(1-\Gamma) - (1-\theta^*(\Gamma))C_r(\Gamma) - \theta^*(\Gamma) \cdot C_p(1-\Gamma) + a_i^*(\Gamma, v') \cdot v'.$$

Increasing differences follows if

$$a_i^*(\Gamma', v')(v' - F^{-1}(1-\Gamma')) - a_i^*(\Gamma, v')(v' - F^{-1}(1-\Gamma)) \geq a_i^*(\Gamma', v)(v - F^{-1}(1-\Gamma')) - a_i^*(\Gamma, v)(v - F^{-1}(1-\Gamma)). \tag{7}$$

The expression $a_i^*(\Gamma, v)(v - F^{-1}(1-\Gamma))$ is increasing in Γ and v, so increasing differences clearly obtain. The discrete and determinate nature of the choice a^*, however, makes it worthwhile and possible to examine all of the cases carefully. Further, the exercise demonstrates that strict increasing differences do not obtain.

From $v' - F^{-1}(1-\Gamma') > v - F^{-1}(1-\Gamma'), v' - F^{-1}(1-\Gamma) > v - F^{-1}(1-\Gamma)$, six cases follow:

1. $0 > v' - F^{-1}(1-\Gamma') > v - F^{-1}(1-\Gamma)$
2. $v' - F^{-1}(1-\Gamma') \geq 0 > v - F^{-1}(1-\Gamma)$
   (a) $0 > v - F^{-1}(1-\Gamma'), v' - F^{-1}(1-\Gamma)$
   (b) $v' - F^{-1}(1-\Gamma) \geq 0 > v - F^{-1}(1-\Gamma')$
   (c) $v - F^{-1}(1-\Gamma') \geq 0 > v' - F^{-1}(1-\Gamma)$
   (d) $v - F^{-1}(1-\Gamma'), v' - F^{-1}(1-\Gamma) \geq 0$
3. $v' - F^{-1}(1-\Gamma') > v - F^{-1}(1-\Gamma) \geq 0$

Equation (7) reduces, by case, to:

1. $0 \cdot (v' - F^{-1}(1-\Gamma')) - 0 \cdot (v' - F^{-1}(1-\Gamma)) \geq 0 \cdot (v - F^{-1}(1-\Gamma')) - 0 \cdot (v - F^{-1}(1-\Gamma))$, which holds weakly because $0 \geq 0$.

2. (a) $(v' - F^{-1}(1-\Gamma')) - 0 \cdot (v' - F^{-1}(1-\Gamma)) \geq 0 \cdot (v - F^{-1}(1-\Gamma')) - 0 \cdot (v - F^{-1}(1-\Gamma))$, which holds strictly because $v' - F^{-1}(1-\Gamma') > 0$.

   (b) $(v' - F^{-1}(1-\Gamma')) - (v' - F^{-1}(1-\Gamma)) \geq 0 \cdot (v - F^{-1}(1-\Gamma')) - 0 \cdot (v - F^{-1}(1-\Gamma))$, which holds strictly because $F^{-1}(1-\Gamma) > F^{-1}(1-\Gamma')$.

   (c) $(v' - F^{-1}(1-\Gamma')) - 0 \cdot (v' - F^{-1}(1-\Gamma)) \geq (v - F^{-1}(1-\Gamma')) - 0 \cdot (v - F^{-1}(1-\Gamma))$, which holds strictly because $v' > v$.

   (d) $(v' - F^{-1}(1-\Gamma')) - (v' - F^{-1}(1-\Gamma)) \geq (v - F^{-1}(1-\Gamma')) - 0 \cdot (v - F^{-1}(1-\Gamma))$, which holds strictly because $0 > v - F^{-1}(1-\Gamma)$

3. $(v' - F^{-1}(1-\Gamma')) - (v' - F^{-1}(1-\Gamma)) \geq (v - F^{-1}(1-\Gamma')) - (v - F^{-1}(1-\Gamma))$, which holds weakly because $0 \geq 0$.

This establishes non-strict increasing differences between Γ and v. ■

Proposition 4. *If the majority-preferred policy intervention does (resp., does not) induce a majority of the population to choose $a = 1$ instead of $a = 0$, then the level of overall compliance will be greater (resp., less) than in the utilitarian optimal policy, regardless of the type of policy used.*

Proof of Proposition 4. Define

$$U^1_{MV}(\Gamma, \theta) = W(\Gamma) + (1-\Gamma) \cdot (-F^{-1}(1-\Gamma)) - (1-\theta) \cdot C_r(\Gamma) - \theta \cdot C_p(1-\Gamma) + v_{MV}$$

and

$$U^0_{MV}(\Gamma, \theta) = W(\Gamma) + \Gamma \cdot F^{-1}(1-\Gamma) - (1-\theta) \cdot C_r(\Gamma) - \theta \cdot C_p(1-\Gamma).$$

Viewing the median voter's action a_{MV} as a parameter, it follows that the (set of) majority-preferred intervention(s) when $a^{MV} = 1$ is greater than the (set of) majority-preferred intervention(s) when $a^{MV} = 0$. To demonstrate that these interventions "sandwich" the utilitarian-optimal interventions, consider the first derivatives with respect to Γ:

$$\begin{array}{rcl} \frac{\partial U^1_{MV}}{\partial \Gamma} & = & W' + \frac{1-\Gamma}{f(F^{-1}(1-\Gamma))} + F^{-1}(1-\Gamma) - C'_\theta \\ & > & \\ \frac{\partial U_{SP}}{\partial \Gamma} & = & W' + F^{-1}(1-\Gamma) - C'_\theta \qquad ,\forall \Gamma \in [\Gamma_0, 1]. \\ & > & \\ \frac{\partial U^0_{MV}}{\partial \Gamma} & = & W' - \frac{\Gamma}{f(F^{-1}(1-\Gamma))} + F^{-1}(1-\Gamma) - C'_\theta \end{array}$$

Viewed in the context of Proposition 1, were we to define $\hat{W}' = W' + \frac{1-\Gamma}{f(1-\Gamma)}$ and $\hat{\hat{W}}' = W' - \frac{\Gamma}{f(1-\Gamma)}$, we would find $\hat{W}' \geq W'$ and $W' \geq \hat{\hat{W}}'$ pointwise. Hence, it is as though the utilitarian's objective function maximizes over a higher (lower) marginal benefit for the $a^{MV} = 1$ ($a^{MV} = 0$) case. The monotone comparative statics established in 1 imply:

$$\arg\max_{(\Gamma,\theta)} U^1_{MV} \geq \arg\max_{(\Gamma,\theta)} U_{SP} \geq \arg\max_{(\Gamma,\theta)} U^0_{MV}.$$

Since a^{**}_{MV} also indicates the action a majority of the population will take, a majority complying will favor interventions with greater than optimal compliance, while a majority not complying will favor interventions with lower than optimal compliance. ■

Proposition 5. *Consider $W, \hat{W}$, and suppose $\hat{W}' > W'$ pointwise. Then $(\Gamma^{**}, \theta^{**})$ and a^{**}_{MV} are weakly larger under $\hat{W}$ than under W.*

Proof of Proposition 5. Recall that $(\Gamma^{**}, \theta^{**})$ denotes the median voter's set of optimal policy interventions and a^{**}_{MV} the optimal action associated with each policy intervention, where $a^{**}_{MV} := a^*_{MV}(\Gamma^{**}, v_{MV})$, $\theta^{**} := \theta^*(\Gamma^{**})$, and Γ^{**} is as in 5 (the set of maximizers of the equation below).

$$U_{MV}(\Gamma) = W(\Gamma) - (a^*_{MV}(\Gamma, v) - \Gamma) \cdot F^{-1}(1-\Gamma) - (1-\theta^*(\Gamma))C_r(\Gamma) - \theta^*(\Gamma) \cdot C_p(1-\Gamma) + a^*_{MV}(\Gamma, v_{MV}) \cdot v_{MV}$$

Precisely the same calculations as found in the proof of Proposition 1 establish the increasing differences of $(\Gamma; W)$. As such, U_{MV} possesses monotone comparative statics in W, according to the specified partial order. ■

Corollary 1. *Let $v_{DV} < \hat{v}_{DV}$. Then $(\Gamma^{**}, \theta^{**})$ and a^{**}_{DV} are weakly larger under $\hat{v}_{DV}$ than under v_{DV}.*

Proof of Corollary 1. Because U_i has increasing differences in (Γ, v_i) – by way of $a^*_i(\Gamma, v_i)$ – Γ^{**} is weakly increasing in v_{DV}, as are $a^{**}_{DV} = a^*_{DV}(\Gamma^{**}, v_{DV})$ and $\theta^{**} = \theta^*(\Gamma^{**})$. ■

B Incorporating Uncertainty

B.1 Incomplete Information about the Location of the Median Valuation

In this subsection, we allow the v_i to be stochastic rather than deterministic to probe the robustness of the importance of the median voter (see Lemma 4). Specifically, we more explicitly consider platform competition between two candidates, asking whether the median voter – or an analogue to it – provides an indication of the policy selected under majoritarian politics. We sketch the result below, but it broadly hews to the "median of medians" result from Calvert (1985) (see also Duggan (2008)).

Suppose two office-motivated candidates compete with binding policy platforms over a continuum of voters i of mass one. Let $v_i \overset{iid}{\sim} F(\cdot), \forall i$. We continue to assume that competition occurs over the optimal size of the intervention, given the optimal type of intervention. Two additional assumptions streamline the exposition. First, limit consideration to active policy interventions, i.e., $P \neq 0$ and/or $R \neq 0$. Second, let the maximizers of the following be single-valued and given by $\overline{\Gamma}^{**}$ and $\underline{\Gamma}^{**}$, respectively:

$$U^1_{MV}(\Gamma) = W(\Gamma) + (1-\Gamma)\cdot(-F^{-1}(1-\Gamma)) - (1-\theta^*(\Gamma))\cdot C_r(\Gamma) - \theta^*(\Gamma)\cdot C_p(1-\Gamma) + v_{MV},$$

$$U^0_{MV}(\Gamma) = W(\Gamma) + \Gamma\cdot F^{-1}(1-\Gamma) - (1-\theta^*(\Gamma))\cdot C_r(\Gamma) - \theta^*(\Gamma)\cdot C_p(1-\Gamma).$$

Per Lemma 4, $\overline{\Gamma}^{**} > \underline{\Gamma}^{**}$, where $\overline{\Gamma}^{**}$ is the most-preferred level of compliance for *ex post* compliers, and $\underline{\Gamma}^{**}$ is the most-preferred level of compliance for *ex post* non-compliers. Compliance decisions are, of course, dependent upon the size of the intervention. As such, if $v_i \geq F^{-1}(1-\overline{\Gamma}^{**}) =: \tilde{v}$, i's most-preferred intervention is $\overline{\Gamma}^{**}$. If $v_i < F^{-1}(1-\overline{\Gamma}^{**}) < F^{-1}(1-\underline{\Gamma}^{**})$, i's most-preferred intervention is $\underline{\Gamma}^{**}$. Let $G(v) = \Pr(v_{MV} \geq v)$.[22] Then $G(\tilde{v})$ is the probability that $\geq 50\%$ of the population will prefer the intervention $\overline{\Gamma}^{**}$.

It follows that candidates choose between $\overline{\Gamma}^{**}$ and $\underline{\Gamma}^{**}$. We may normalize the office-holding benefit to 1. If the probability with which ties are broken in favor of each candidate is 1/2, and we suppose the probability that a candidate's opposition chooses $\overline{\Gamma}^{**}$ is given by q, then a candidate chooses $\overline{\Gamma}^{**}$ if

$$\begin{aligned} EU(\overline{\Gamma}^{**}) &\geq EU(\underline{\Gamma}^{**}) \Leftrightarrow \\ G(\tilde{v})(q/2+(1-q)) + (1-G(\tilde{v}))(q/2) &\geq G(\tilde{v})((1-q)/2) + (1-G(\tilde{v}))(q+(1-q)/2) \Leftrightarrow \\ q/2 + G(\tilde{v})(1-q) &\geq (1-q)/2 + (1-G(\tilde{v}))q \Leftrightarrow \\ G(\tilde{v}) &\geq 1/2. \end{aligned}$$

The final line indicates that it is a dominant strategy for the candidates to converge to the preferred policy of the "estimated median" (Calvert 1985), which in this case refers to the median order statistic. With a continuum (or taking the limit of n draws as $n \uparrow \infty$), the median order statistic converges to the median of the parent distribution with variance tending to zero. If $\overline{\Gamma}^{**} \geq 1/2$, then both candidates converge in equilibrium to propose $(\overline{\Gamma}^{**}, \theta^*(\overline{\Gamma}^{**}))$ as their platform, otherwise $(\underline{\Gamma}^{**}, \theta^*(\underline{\Gamma}^{**}))$, where $\theta^*(\underline{\Gamma}^{**}) \leq \theta^*(\overline{\Gamma}^{**})$.

B.2 Allowing for Imperfect Enforcement

Let the probability of an individual who takes $a = 0$ being mistaken for having taken $a = 1$ ("type I error") be denoted α and the probability that an individual who took $a = 1$ being mistaken for having taken $a = 0$ ("type II error") be denoted β, with $\alpha, \beta < 1/2$. A member of the population i then chooses $a = 1$ if

[22] Where the identity of MV is determined after the v_i are realized and platforms proposed.

$(1-\beta)R - \beta \cdot P + v_i \geq \alpha \cdot R - (1-\alpha)P \Rightarrow v_i \geq -(1-\beta-\alpha)(P+R)$.

The proportion of the population complying is then $1 - F(-(1-\alpha-\beta)(P+R))$, still denoted Γ. Letting $X = P + R$, we have $X(\Gamma) = -F^{-1}(1-\Gamma)/(1-\alpha-\beta)$.

The proportion of the population paying a fine P is $(1-\alpha)(1-\Gamma) + \beta\Gamma = (1-\alpha) - (1-\alpha-\beta)\Gamma$, so the cost of fines is $C_p((1-\alpha) - (1-\alpha-\beta)\Gamma)$.

The proportion of the population receiving a subsidy R is $\alpha(1-\Gamma) + (1-\beta)\Gamma = \alpha + (1-\alpha-\beta)\Gamma$, so the cost of subsidies is $C_r(\alpha + (1-\alpha-\beta)\Gamma)$.

The social benefit and sum of valuations for those choosing $a = 1$ remain unchanged as functions of Γ.

The incremental return of using punishments instead of rewards

$$U_{SP}(\Gamma, p) - U_{SP}(\Gamma, r) = C_r(\alpha + (1-\alpha-\beta)\Gamma) - C_p((1-\alpha) - (1-\alpha-\beta)\Gamma).$$

Taking the derivative with respect to Γ, we have $(1-\alpha-\beta)(C_r' + C_p') > 0$, so we continue to have increasing differences in (Γ, θ).

Taking the derivative instead with respect to α, we have $(1-\Gamma)(C_r' + C_p')$, so we have increasing differences in (θ, α). If taken with respect to β, we have $-\Gamma(C_r' + C_p')$ and decreasing differences with respect to (θ, β).

Considering (Γ, α):

$$\begin{aligned} \frac{\partial^2}{\partial\Gamma\partial\alpha} U_{SP}(\Gamma, p) &= \frac{\partial}{\partial\Gamma} - (1-\Gamma)C_p'((1-\alpha) - (1-\alpha-\beta)\Gamma) \\ &= C_p' + (1-\alpha-\beta)(1-\Gamma)C_p'' \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{\partial^2}{\partial\Gamma\partial\alpha} U_{SP}(\Gamma, r) &= \frac{\partial}{\partial\Gamma}(1-\Gamma)C_r'(\alpha + (1-\alpha-\beta)\Gamma) \\ &= -C_r' + (1-\alpha-\beta)(1-\Gamma)C_r'' \end{aligned} \tag{9}$$

Considering (Γ, β):

$$\begin{aligned} \frac{\partial^2}{\partial\Gamma\partial\beta} U_{SP}(\Gamma, p) &= \frac{\partial}{\partial\Gamma}\Gamma C_p'((1-\alpha) - (1-\alpha-\beta)\Gamma) \\ &= C_p' - (1-\alpha-\beta)\Gamma C_p'' \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{\partial^2}{\partial\Gamma\partial\beta} U_{SP}(\Gamma, r) &= \frac{\partial}{\partial\Gamma} - \Gamma C_r'(\alpha + (1-\alpha-\beta)\Gamma) \\ &= -C_r' - (1-\alpha-\beta)\Gamma C_r'' \end{aligned} \tag{11}$$

If the cost functions convex in addition to increasing, then (8) and (11) are, respectively, unambiguously positive and negative. If (9) and (10) are, respectively, positive and negative, then we may conclude that the social planner's objective function possesses monotone comparative statics in α and $-\beta$. The proposition below follows as long as, for all $\Gamma \in (\Gamma_0, 1)$, both of the following conditions hold:

$$\frac{C_r''(\alpha + (1-\alpha-\beta)\Gamma)}{C_r'(\alpha + (1-\alpha-\beta)\Gamma)} \geq \frac{1}{(1-\alpha-\beta)(1-\Gamma)} \ , \tag{12}$$

$$\frac{C_p''((1-\alpha) - (1-\alpha-\beta)\Gamma)}{C_p'((1-\alpha) - (1-\alpha-\beta)\Gamma)} \geq \frac{1}{(1-\alpha-\beta)\Gamma} \ . \tag{13}$$

Proposition 6. *Letting α be the probability that an individual choosing $a = 0$ is mistakenly attributed the action of $a = 1$ and β be the probability that an individual choosing $a = 1$ is mistakenly attributed the action of $a = 0$, the social planner's objective function displays monotone comparative statics in α and $-\beta$ as long as $C_r(\cdot)$ and $C_p(\cdot)$ are sufficiently convex, i.e., satisfy* (12) *and* (13)*.*

Proof of Proposition 6. The proof follows immediately from Lemma 7 and the increasing differences in (Γ, θ), (θ, α), and (Γ, α), as well as $(\theta, -\beta)$, and $(\Gamma, -\beta)$. ■

As the probability α of incorrectly classifying a member of the population that chose $a = 0$ as having chose $a = 1$ increases, punishments become less costly to administer, all else equal. One direct effect is to make the use of punishments more attractive relative to the use of rewards. While cheaper punishments can encourage the social planner to allow greater non-compliance, the supposition of sufficient convexity ensures that costs rise too quickly as non-compliance increases, thus putting upward pressure on the optimal share choosing $a = 1$ *ex post*. The complementarity between using punishments and inducing a high share of the population to choose $a = 1$ *ex post* only reinforce the direct effects. Increases in α then lead to an increase in the optimal share of the population choosing $a = 1$ *ex post*, favoring the use of punishments. An analogous logic describes how increases in the probability of classifying an individual choosing $a = 1$ as one who took $a = 0$ lead to lower optimal levels of compliance and favor the use of rewards instead of punishments as the optimal type of policy.

C Accounting for Unaffected Subpopulations

The model has thus far set aside the possibility that the policy applies only to a "subpopulation of interest," with the population at-large not confronted with a choice between $a = 1$ and $a = 0$. This may take one of two forms: 1) it is not possible to distinguish the subpopulation of interest from the rest of the population for the sake of applying the policy, 2) it is possible to administer the policy only to the subpopulation of interest. The former receives attention first, and it requires only an informal discussion as an application of earlier results covers this case without difficulty.

When the subpopulation of interest is indistinguishable from the population at-large and the subpopulation not of interest tacitly chooses $a = 1$ (e.g., non-drivers not speeding), it is as though the distribution of valuations for the whole population is a first-order stochastic increase of the distribution of valuations of the subpopulation of interest (drivers). Invoking Proposition 1(b), this favors the use of disincentives. In the example of discouraging speeding, this formalizes the intuition that rewarding a substantial segment of the population (non-drivers) for not speeding (when they were at no risk of doing so anyway) would be incredibly inefficient. From the utilitarian's perspective, these conditions favor the use of fines and inducing a high share of the population choosing to drive safely (or not at all), and the fines would likely be even larger if majority-preference dictated the choice of policy.

In the context of copyright for artistic works, or patents for inventions, government wishes to encourage innovation, but it is unable to target the subpopulation of possible innovators, artistic or otherwise. This constitutes the presence of a large subpopulation that is not disposed to "comply" with the behavior government wishes to encourage. As such, the distribution of valuations of the population as a whole is less prone to choose $a = 1$ than the distribution of valuations within the subpopulation of interest. Referencing Proposition 1(b) again, these conditions lead a utilitarian to favor using rewards to spur innovation by a small share of the overall population. The implies the majority-preference would be for a smaller-than-optimal reward, which foots with oft-heard complaints from innovators across fields, namely, that the reward is insufficient compensation for their creative effort.

In many circumstances, however, it is easy to differentiate those in a subpopulation of interest from those who are not. For instance, a policymaker may wish to target an industry. It is usually straightforward to identify firms from individuals and, further, firms in a certain industry from firms in other industries. Those who do not own cars would not be penalized for failure to possess vehicle registration, and those without cropland would be ineligible for farm subsides. Call the portion of the population that would receive neither a reward nor a punishment under a given policy the "unaffected subpopulation." The "subpopulation of interest" still refers to the portion of the population to whom any incentive or disincentive would apply.

If enforcement is able to discriminate between the subpopulation of interest and the rest of the population, then the analyses of the utilitarian's optimal policy hold without further modification. The policymaker may ignore redistributive implications for subpopulations not directly affected by the policy, as she could with redistributive implications for individuals in the subpopulation affected by the policy. In the analysis of the popular support for incentive and disincentive policies, however, the presence of an unaffected subpopulation will materially affect the analysis. The unaffected subpopulation certainly reaps social benefit from compliance with the desired behavior in the subpopulation of interest. Furthermore, members of the unaffected subpopulation must also contribute to the financing of subsidies, but they may likewise benefit from the redistribution of fines or taxes collected.

Let the size of the subpopulation not directly affected by the policy, i.e., not eligible for a reward or punishment, be given by $\lambda \in (0,1)$. Denote an arbitrary member of this group by ℓ. The entire population is still of mass 1, so the size of the subpopulation of interest is of mass $1-\lambda$.[23]

Under a policy that involves the use of rewards (as well as potentially punishments), ℓ would have to contribute $(1-\lambda)\cdot R\cdot(1-F(-R-P))+C_r((1-\lambda)\cdot[1-F(-R-P)])$ to finance the subsidy, but receive no compensation for her behavior other than social benefit given by $W((1-\lambda)\cdot[1-F(-R-P)])$. With regards to policies applied to those choosing $a=1$, then, ℓ's utility function takes the same form as a member of the subpopulation of interest who chooses $a=0$. In contrast, under a punishment-based policy, ℓ will not receive any fine, but she will receive $(1-\lambda)\cdot P\cdot F(-R-P)-C_p((1-\lambda)\cdot F(-R-P))$ and $W((1-\lambda)\cdot[1-F(-R-P)])$. Thus, with regards to policies applied to those choosing $a=0$, ℓ shares the same utility function as a member of the subpopulation of interest who chooses $a=1$. Accordingly, maximizing ℓ's utility entails comparing the most-preferred punishment-based policy for a member of the sub-population of interest who *ex post* chooses $a=1$ to the most-preferred reward-based policy for a member of the sub-population of interest who *ex post* chooses $a=0$, and then comparing the best of those to the utility ℓ receives in the absence of any further policy intervention, namely $W((1-\lambda)(1-F(0)))$.

The result below characterizes the preferences of a member of an unaffected subpopulation, focusing on the utilitarian-optimal policy intervention and the majority-preferred policy intervention. Assume that the share of the population that is unaffected by the policy is given by $\lambda \in (\frac{1}{2},1)$, such that a majority of the population will receive neither incentive nor disincentive under a policy intervention.

Corollary 2. *When the unaffected subpopulation is larger than the subpopulation of interest, majority-preferred policy will entail either a larger-than-optimal disincentive applied to those choosing $a=0$ or a smaller-than-optimal incentive applied to those choosing $a=1$ (potentially a "negative reward," punishing those who choose $a=1$).*

The optimal policy displays monotone comparative statics with respect to pointwise increases in the marginal benefit to society of compliance.

Proof of Corollary 2. The arguments above establish that the majority-preferred policy intervention will be the more preferred of the complier-preferred disincentive and the non-complier-preferred incentive. Moreover, as the $(1-\lambda)$ drops out of the first-order condition, these policies coincide with the majority-preferred policies in the prior section. The comparative statics of the optimal policy intervention then follow immediately from Proposition 5. ∎

[23] As above, it is never the case that $P \neq 0$ and $R \neq 0$.

When a subpopulation not directly affected by incentives or disincentives in a given policy domain is sufficiently large so as to decide the policy for the entire population, it will always be the case that rewards will achieve smaller than the socially optimal share of the population choosing $a = 1$ and punishments will achieve larger than the socially optimal share of the population choosing $a = 1$. This was the tendency noted in Proposition 4, but incorporating the unaffected subpopulation makes this a certainty.

An increase in the marginal benefit to society of members of the affected subpopulation choosing $a = 1$ rather than $a = 0$ increases the utility ℓ receives from P^{**} and decreases the utility ℓ receives from R^{**}. A similar statement holds for first-order stochastic increases in the distribution of valuations for choosing $a = 1$ rather than $a = 0$ among the subpopulation of interest. The downward pressure on rewards may drive them to become negative, constituting a disincentive for choosing $a = 1$.

The onerous fines that farms incur to receive organic certification are an example of a negative reward applied to those in a small subpopulation of interest that take a socially beneficial behavior. The low marginal benefit of a given (usually small) farm choosing to adopt organic practices favors the use of policies applied to those choosing $a = 1$, which further leads to smaller-than-optimal (even negative) incentives. That such a policy would generate revenue to be redistributed among the population-at-large would only help to overcome the loss of social benefit for a member of the unaffected subpopulation.